THE NATIONAL INSURANCE INSTITUTE
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INCOME TAX EVASION AND THE SUPPLY OF LABOR

by
Gideon Yaniv

DISCUSSION PAPER 14

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I. INTRODUCTION*

Income tax evasion as an illegal activity which involves risk has begun to attract theoretical interest since Allingham and Sandmo (1972) and Srinivisan (1973) introduced models for determining a taxpayer's optimal extent of evading taxes. Assuming the level of activity in the labor market to be given, both models reduce the individual's problem into that of determing the optimal under-reporting of a given income. One may suspect however, that the problems of allocating time to work and under-reporting its income are mutually dependent, so that the individual's decisions are obtained rather simultaneously. Hence there would be no a priori guarantee, for example, that an effective campaign against tax evasion might not lead eventually to a reduction in the supply of work effort and government's receipts as well.

Recognizing this, the present model explores some connections between labor supply and tax evasion for a risk averse taxpayer. The nature of the individual's equilibrium is considered in the first section. The following section develops a comparative static analysis, generalizing Allingham and Sandmo's discussion of the simple case. Two main questions are considered in the subsequent sections: Whether evading taxes is accompanied by an increased or diminished labor supply, and, secondly, under the constraint of equal governments' receipts, whether the total level of work in a tax evasion economy would exceed or fall short of that taking place in a non-evasion economy. The main results are summarized in the last section.

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* I wish to thank Michael Bruno who has provided me with many valuable comments and suggestions. I am also indebted to Giora Hanoch, Marjorie Honig and especially to Nehemia H. Shiff for helpful advice and discussions.

II. THE MODEL[1]

Let us assume that the individual faces a constraint of G hours that can be allocated between leisure and work in a given period. The market wage rate per hour, w, and the income tax rate, $0 < t < 1$, are given to him exogenously, independent of the number of hours, g, he chooses to work. The individual has also the choice between two strategies: The first is to declare his actual income, wg, so that his disposable income, $I_d^{\,0}$, will be

$$I_d^{\,0} = (1 - t)\, wg \tag{1}$$

The second is to declare less than his actual income, $\hat{I}$, exposing himself to a possible investigation by the tax authorities who may submit his case to court. His probability of conviction, p, will be assumed to depend upon the resources and efficiency of the tax authorities, thus given to him exogenously.[2] The latter strategy creates for the individual two possible states of the world; If he is not convicted, his disposable income, $I_d^{\,nc}$, will be

$$I_d^{\,nc} = wg - t\hat{I} = (1 - \epsilon t)\, wg \tag{2}$$

where $0 < \epsilon < 1$ denotes the proportion of actual income declared, $\frac{\hat{I}}{wg}$. In this case, the individual succeeds in under-reporting his actual income by the sum of $(1 - \epsilon)$ wg, thus evading taxes of $(1 - \epsilon)$ twg. Let us follow Yitzhaki (1974) in assuming that in case of a conviction the individual expects a penalty in a form of a monetary fine which is proportional to the level of taxes he evaded. To make his choise a non-trivial one, the penalty rate, b, should be assumed to exceed unity.[3] That is, the penalty function, F, is given by

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1 Most of the tedious differentiating and rearranging work involved in achieving results for this model is excluded from what follows, and is available from the author upon request.

2 A more realistic assumption, but somewhat inconvenient theoretically, would allow the probability of conviction to be a function of reported income, i.e. $p = p(\hat{I})$. For a discussion of this case see Allingham and Sandmo.

3 As an alternative one may adopt Allingham's and Sandmo's assumption that the fine is proportional to the level of unreported income by a rate which exceeds t. However, Yitzhaki's assumption is more convenient in labor supply analysis since it neutralizes evasion profitability from a dependence upon the income tax rate, thus simplifying the following results.

$$F = b(1-\epsilon)\,twg \tag{3}$$

where $b > 1$.

In case of conviction, the individual's disposable income, $I_d^{\,c}$, will thus be

$$I_d^{\,c} = [\,1 - t(\epsilon + b(1-\epsilon))\,]\,wg \tag{4}$$

The individual now chooses ϵ^* and g^* so as to maximize his expected utility defined upon disposable income and leisure

$$EU(I_d, G-g) = (1-p)\,U(I_d^{\,nc}, G-g) + pU(I_d^{\,c}, G-g) \tag{5}$$

The utility function satisfies the common properties of positive and decreasing marginal utilities in both arguments. To enable the evaluation of the following results in terms of the well-known risk aversion measures it will be assumed that the utility function is additive in income and leisure[4] so that (5) can be written as

$$EU(I_d, G-g) = (1-p)\,U(I_d^{\,nc}) + pU(I_d^{\,c}) + V(G-g) \tag{6}$$

and the individual is risk averse by assumption [$U''(I_d) < 0$].

Maximization of (6) subject to (2) and (4) yields the following two first-order conditions for an interior solution for ϵ^* and g^*, respectively

$$\frac{\partial EU}{\partial \epsilon} = -(1-p)\,U'(I_d^{\,nc})\,twg + pU'(I_d^{\,c})\,(b-1)\,twg = 0 \tag{7}$$

$$\frac{\partial EU}{\partial g} = (1-p)\,U'(I_d^{\,nc})\,(1-\epsilon t)\,w + pU'(I_d^{\,c})\,[\,1 - t(\epsilon + b\,(1-\epsilon))\,]\,w - V'(G-g) = 0 \tag{8}$$

[4] Note that the additivity of the utility function does not imply independency between the decisions to evade taxes and to allocate time to work, since both ϵ and g appear in $U(I_d^{\,nc})$ and $U(I_d^{\,c})$.

where the following second-order conditions for a maximum are fulfilled under the assumptions on the utility function[5]

$$D \equiv \frac{\partial^2 EU}{\partial \epsilon^2} = (1-p)\,U''(I_d^{nc})\,(twg)^2 + pU''(I_d^{c})\,[\,(b-1)\,twg]^2 < 0 \qquad (9)$$

$$B \equiv \frac{\partial^2 EU}{\partial g^2} = (1-p)\,U''(I_d^{nc})\,(1-\epsilon t)^2 w^2 + pU''(I_d^{c})\,[\,1 - t(\epsilon + b(1-\epsilon))\,]^2 w^2 + V''(G-g) < 0 \qquad (10)$$

Rearranging (7) and (8), the optimum conditions for ϵ^* and g^* can be written, respectively, as

$$\frac{U'(I_d^{nc})}{(b-1)\,U'(I_d^{c})} = \frac{p}{1-p} \qquad (11)$$

$$\rho^{nc}\,(1-\epsilon t)w + \rho^{c}\,[\,1 - t(\epsilon + b(1-\epsilon))\,]\,w = \frac{V'(G-g)}{EU'(I_d)} \qquad (12)$$

where $\rho^i = \dfrac{p^i U'(I_d^{\,i})}{EU'(I_d)}; \quad \sum_{i=nc,c} \rho^i = 1$

Condition (11) implies that the optimal proportion declared (for a given level of work) should be determined such that the ratio between the "values" of the marginal utilities from the two possible states of the world equals the adqeuate odds for their occurrence. Condition (12) implies that the optimal number of hours allocated to work (for a given proportion declared) should be determined such that the marginal rate of substitution between income and leisure in expected utility equals an average of alternative leisure prices weighted by the "shares" of alternative marginal utilities from the two possible states of the world.

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[5] The additional second-order condition, i.e. $\dfrac{\partial^2 EU}{\partial \epsilon^2} \cdot \dfrac{\partial^2 EU}{\partial g^2} > \left(\dfrac{\partial^2 EU}{\partial \epsilon \partial g}\right)^2$ is assumed to hold.

The *optimal conditions* (11) *and* (12) *are diagramatically shown in the following* two figures. Figure 1 demonstrates the determination of ϵ^* for a given g by a tangency of an expected utility locus $\left\{ (1-p)\, U\, (I_d^{\,nc}, G-g) + pU\, (I_d^{\,c}, G-g) = \bar{U} \right\}$ to the curve of utility combinations from the two possible states of the world, UU, resulting from changes in the proportion of actual income declared. The figure emphasizes the necessary conditions for an interior solution: The individual will declare less than his actual income if the slope of UU in the full-report case ($\epsilon = 1$) exceeds the odds for the occurrence of the two possible states of the world. That is, if

$$\frac{1}{b-1} > \frac{p}{1-p} \tag{13}$$

(13) implies that evasion will take place if $bp < 1$. That is, a certain attitude toward risk is not required as an incentive for tax evasion, but sufficiently low values for the government deterrent parameters such that the expected penalty rate is less than unity.

On the other hand, for tax evasion not to be total ($\epsilon = 0$), it is required that

$$\frac{1}{b-1} \cdot \frac{U'[wg]}{U'\,[\,(1-bt)\,wg]} < \frac{p}{1-p} \tag{14}$$

Given the level of work, g, and parameter values for w, t, b, p that satisfy (13) and (14) the concavity assumption on the utility function assures the existence of an interior solution for ϵ^*.

Figure 2 demonstrates the determination of g* for a given ϵ, where for each g the function J (g) is defined to be the disposable income that satisfies

$$U\,[\,J\,(g), G-g] = EU\,(I_d, G-g) \tag{15}$$

The optimal level of work is thus determined by a tangency of an indifference curve to the "budget line", JJ, resulting from changes in labor hours for a given ϵ.[6] A movement

[6] In this formulation the individual maximizes $U\,(I_d, G-g)$ subject to $I_d = J(g)$ for a given ϵ. By the additivity assumption on U, (15) implies $U\,[\,J(g)\,] = EU(I_d)$, so that $U'J' = (1-p)\,U'(I_d^{\,nc})\,(1-\epsilon t)\,w + pU'(I_d^{\,c})\,[\,1 - t\,(\epsilon + b\,(1-\epsilon)\,)\,]\,w$. On the other hand, the optimum condition for this problem is $U'J' = V'$, which is clearly identical to (8).

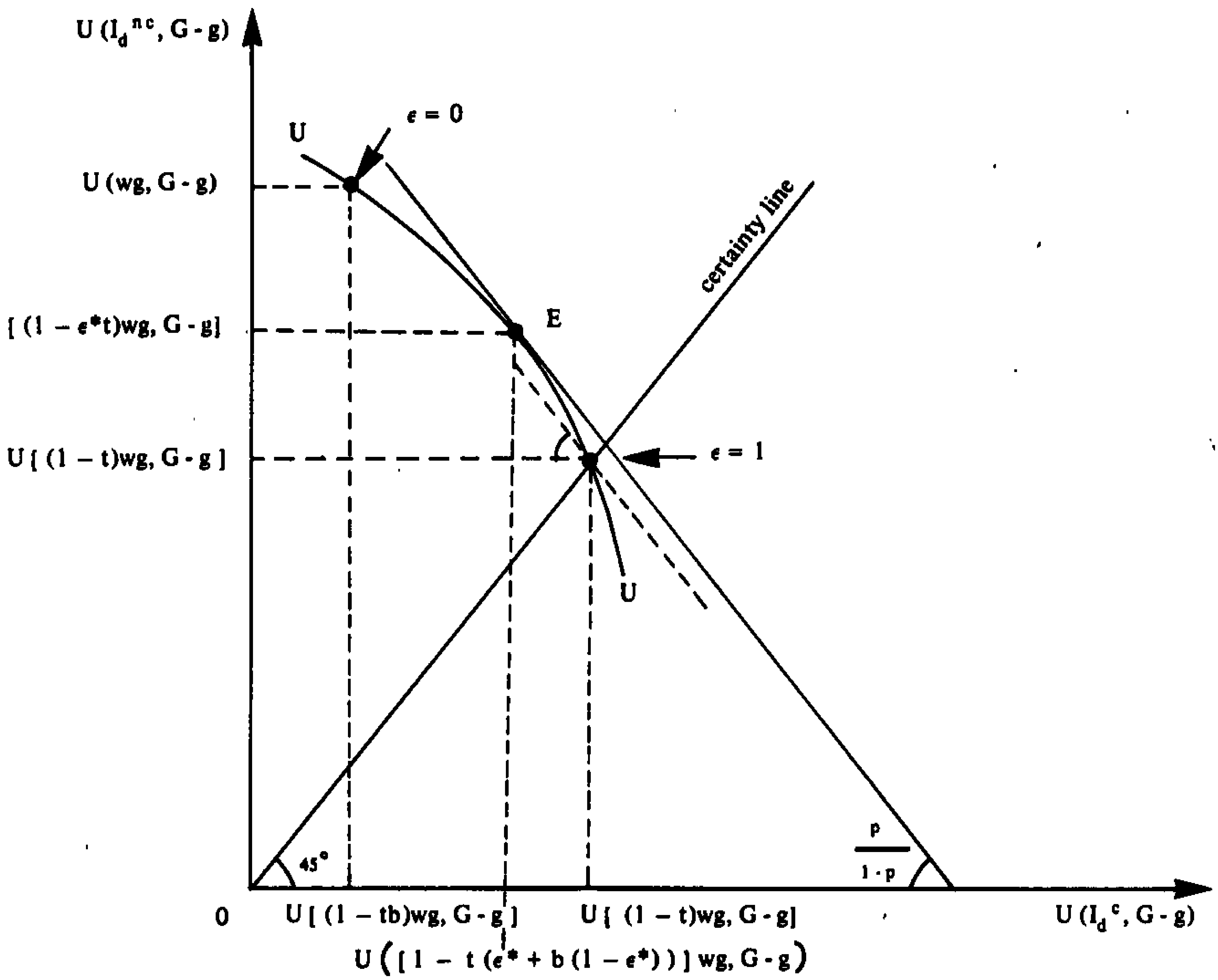


Figure 1: Determination of the proportion of actual income declared (ϵ^*) for a given level of work.

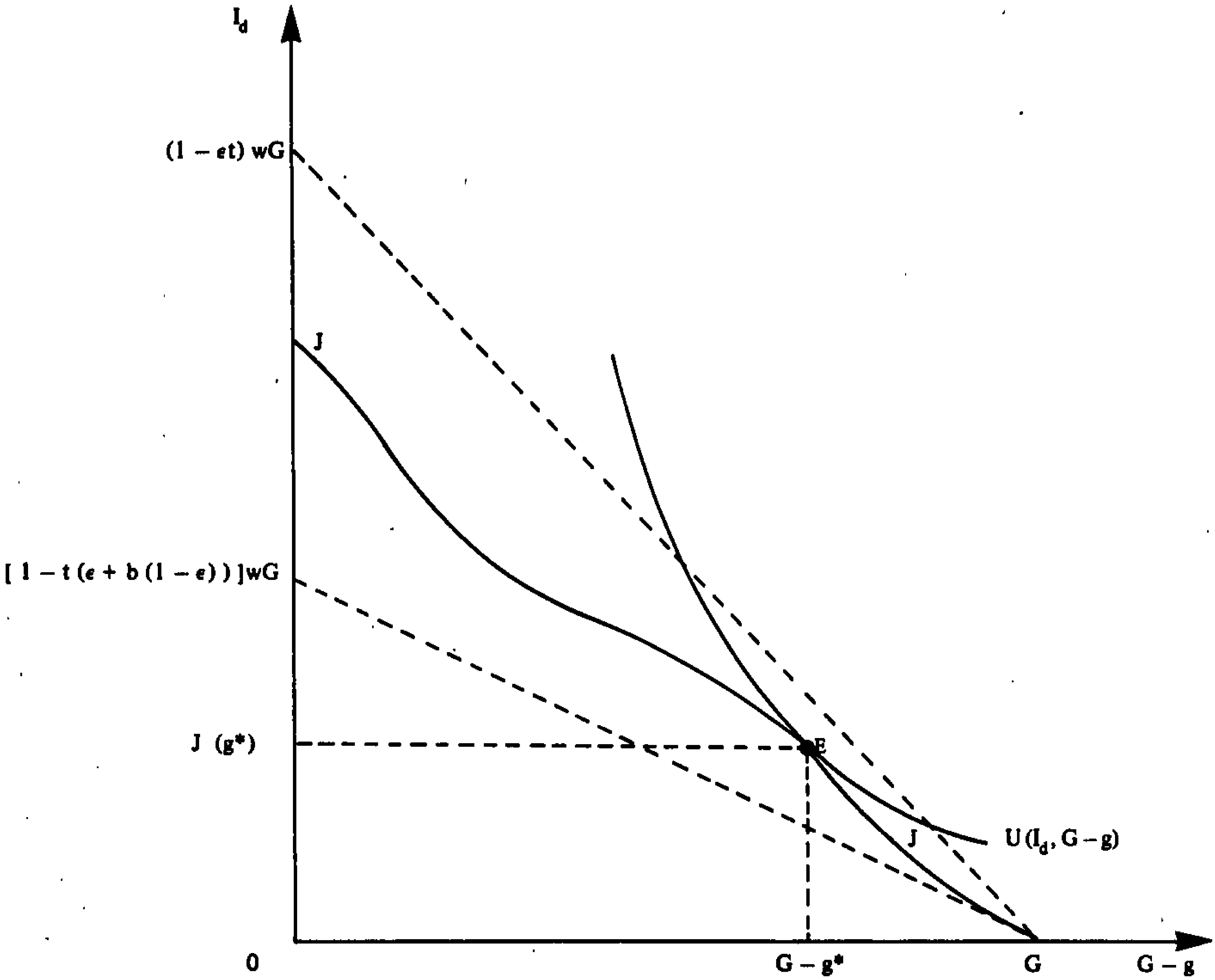


Figure 2 Determination of the level of work (g^*) for a given declared proportion of actual income.

along JJ in Figure 2 represents a shift of the UU curve in Figure 1 (a change in g), while a movement along UU in Figure 1 reflects a shift of the JJ curve in Figure 2 (a change in ϵ). Note that for low levels of g, an increase in the hours of work will cause an outward shift of UU in Figure 1 as long as the reduction in utility from leisure is more than compensated by the increase in utility from disposable income. The inversion of the effects is not independent of ϵ, so that each combination on UU will shift inward in a different level of g. Hence a simultaneous determination of ϵ^* and g^* from Figure 1 is possible only when the utility possibilities frontier is given. In the same way, the simultaneous solution is not obtained by Figure 2 unless the JJ frontier is given. Note that the two opposite effects that determine the JJ frontier are the expected penalty versus the tax evaded.

For a closer examination of the individual's equilibrium let us express ϵ^* of condition (11) as a function of g and the parameters of the model denoted hereafter by the vector π

$$\epsilon^* = \epsilon\,(g, \pi) \tag{16}$$

while g^* is similarly expressed as a function of ϵ and π

$$g^* = g\,(\epsilon; \pi) \tag{17}$$

Let us consider first the effect of a change in the individual's hours of work on his preferred proportion of income declared. Differentiating (11) with respect to g yields

$$\frac{\partial \epsilon^*}{\partial g} = \frac{(b \quad 1)\, twpU'(I_d^{\,c})}{D}\ [\, R_R\,(I_d^{\,c}) - R_R\,(I_d^{\,nc})\,] \tag{18}$$

where $D < 0$ is given by (9) and $R_R\,(I_d) = -\dfrac{U''(I_d)}{U'(I_d)}\, I_d$ is the relative risk aversion measure defined by Arrow and Pratt (1970). It is thus clear from (18) that

$$R_R\,(I_d^{\,nc}) \gtreqless R_R\,(I_d^{\,c}) \iff \frac{\partial \epsilon^*}{\partial g} \gtreqless 0 \tag{19}$$

Hence, the adjustment of the individual's tax evasion to changes in his hours of work depends upon his risk aversion properties. An increase in the level of work (which indicates a rise in disposable income) will increase, leave constant, or decrease the optimal proportion of actual income declared if the relative risk aversion is an increasing,constant or decreasing function of disposable income.

In a similar way we can examine how a change in the individual's proportion of income declared affects his preferred level of work. Differentiating (12) with respect to ϵ yields

$$\frac{\partial g^*}{\partial \epsilon} = \frac{wt}{B} \left\{ (1-p)\, U'(I_d^{nc}) [1 - R_R(I_d^{nc})] - p U'(I_d^{c}) (b-1) [1 - R_R(I_d^{c})] \right\} \qquad (20)$$

where $B < 0$ is given by (10). It is thus obvious that the sign of (20) depends upon the following assumptions on $R_R(I_d)$:

(a) Constant Relative Risk Aversion: $R_R(I_d^{nc}) = R_R(I_d^{c})$

Under this assumption, (20) can be written as

$$\frac{\partial g^*}{\partial \epsilon} = \frac{wt}{B} [1 - R_R(I_d)] [(1-p)\, U'(I_d^{nc}) - (b-1)\, p U'(I_d^{c})] \qquad (21)$$

so that

$$\text{if } R_R(I_d) < 1 \text{ then } \frac{U'(I_d^{nc})}{(b-1)\, U'(I_d^{c})} \gtrless \frac{p}{1-p} \iff \frac{\partial g^*}{\partial \epsilon} \lesseqgtr 0 \qquad (22)$$

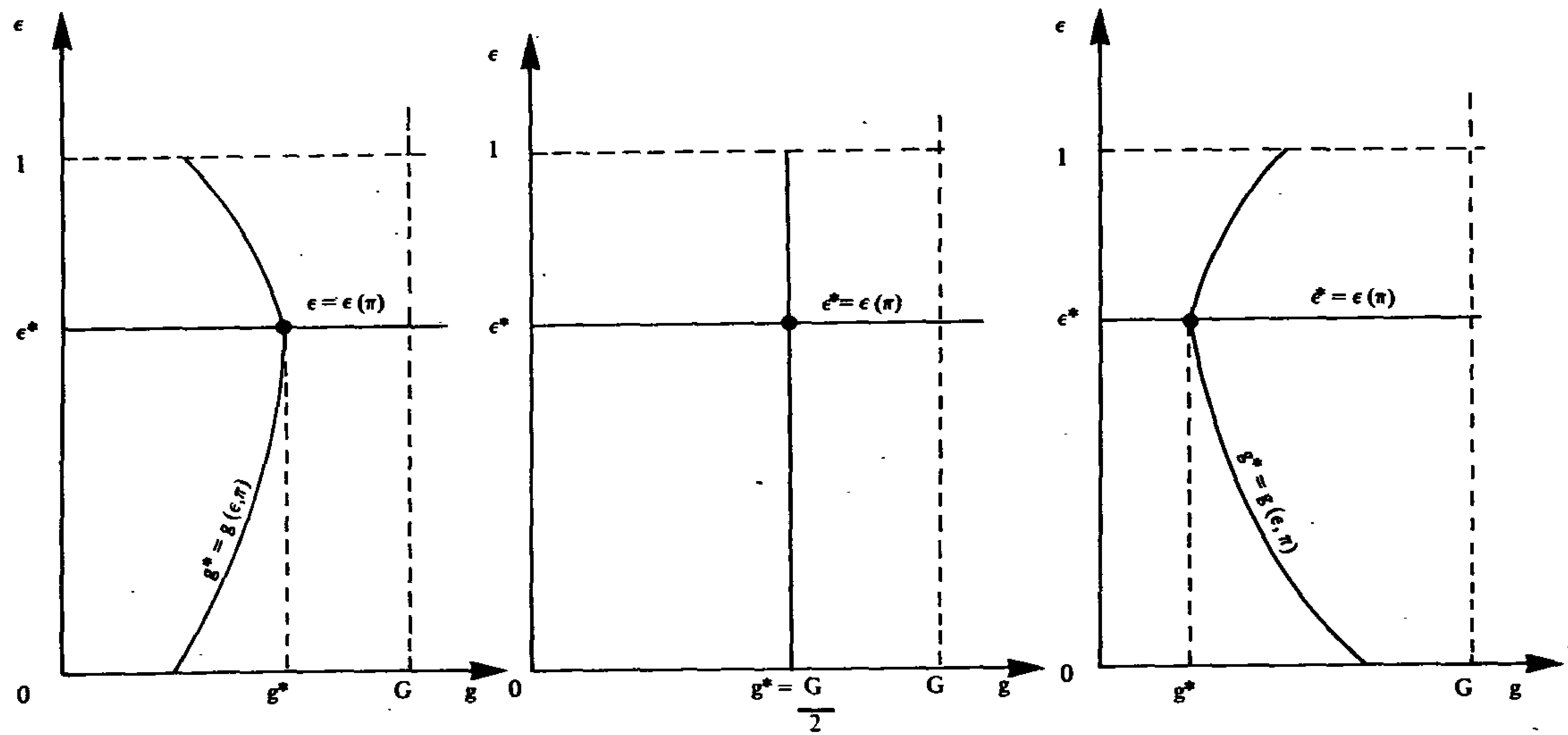
$$\text{if } R_R(I_d) = 1 \text{ then } \frac{\partial g^*}{\partial \epsilon} = 0 \qquad (23)$$

$$\text{and if } R_R(I_d) > 1 \text{ then } \frac{U'(I_d^{nc})}{(b-1)\, U'(I_d^{c})} \gtrless \frac{p}{1-p} \iff \frac{\partial g^*}{\partial \epsilon} \gtrless 0 \qquad (24)$$

The individual's equilibrium for a given π can be now shown in Figure 3: It follows from (19) that for constant relative risk aversion ϵ^* is independent of g and is solely determined by the parameters of the model. Substituting (11) into (21) reveals that the corresponding g^* always satisfies $\left.\frac{\partial g^*}{\partial \epsilon}\right|_{\epsilon = \epsilon^*} = 0.$

It is easily seen from Figure 1 that for $\epsilon > \epsilon^*$ one has $\frac{U'(I_d^{nc})}{(b-1)\, U'(I_d^{c})} > \frac{p}{1-p}$ so it

Figure 3: Determination of the individual's equilibrium of ϵ^* and g^* for constant relative risk aversion.



$R_R(I_d) < 1$ .1

$R_R(I_d) = 1$ .2

$R_R(I_d) > 1$ .3

follows from (22)–(24) that $R_R(I_d) \lesseqgtr 1 \iff \frac{\partial g^*}{\partial \epsilon} \lesseqgtr 0$.

Similarly, for $\epsilon < \epsilon^*$ the opposite inequality holds so that $R_R(I_d) \lesseqgtr 1 \iff \frac{\partial g^*}{\partial \epsilon} \gtreqless 0$.

(b) Increasing Relative Risk Aversion: $R_R(I_d^{nc}) > R_R(I_d^{c})$

Under this assumption, changes in ϵ affect the relative risk aversion measure through their impact on disposable income. To determine the sign of $\frac{\partial g^*}{\partial \epsilon}$, it is first necessary to identify the value of this measure in the full report case, $R_R(I_d^{o})$. If the values of w and t are such so that $R_R(I_d^{o}) < 1$, then for $\epsilon = 1$ we have $1 - R_R(I_d^{nc}) = 1 - R_R(I_d^{c}) > 0$.

Thus, if evasion is profitable [condition (13) is satisfied], then (20) yields $\frac{\partial g^*}{\partial \epsilon} < 0$. When ϵ is reduced, I_d^{c} decreases while I_d^{nc} increases, and by the assumptions on the utility function, the right hand term of (20) increases while the left hand term decreases. For sufficiently small values of ϵ, we thus have $\frac{\partial g^*}{\partial \epsilon} > 0$.

By similar considerations it can be argued that if $R_R(I_d^{o}) \geq 1$, then $\frac{\partial g^*}{\partial \epsilon} > 0$ for each $\epsilon < 1$. Nevertheless, substituting (11) into (20) yields

$$\left.\frac{\partial g^*}{\partial \epsilon}\right|_{\epsilon = \epsilon^*} = \frac{(b-1)\,twpU'(I_d^{c})}{B}\,[\,R_R(I_d^{c}) - R_R(I_d^{nc})\,] \tag{25}$$

so that under this assumption of increasing relative risk aversion the simultaneous optimum must satisfy $\left.\frac{\partial g^*}{\partial \epsilon}\right|_{\epsilon=\epsilon^*} > 0$. Figure 4 illustrates these findings, emphasizing that a a stable equilibrium requires the intersection of $\epsilon(g,\pi)$ from above by $g(\epsilon,\pi)$, satisfying $\frac{\partial \epsilon^*}{\partial g} < 1\Big/\frac{\partial g^*}{\partial \epsilon}$ at this point. Considering (18) and (25) it is thus required that

$$\left\{ (b-1)\,twpU'(I_d^{c})\,[\,R_R(I_d^{c}) - R_R(I_d^{nc})\,] \right\}^2 < BD$$

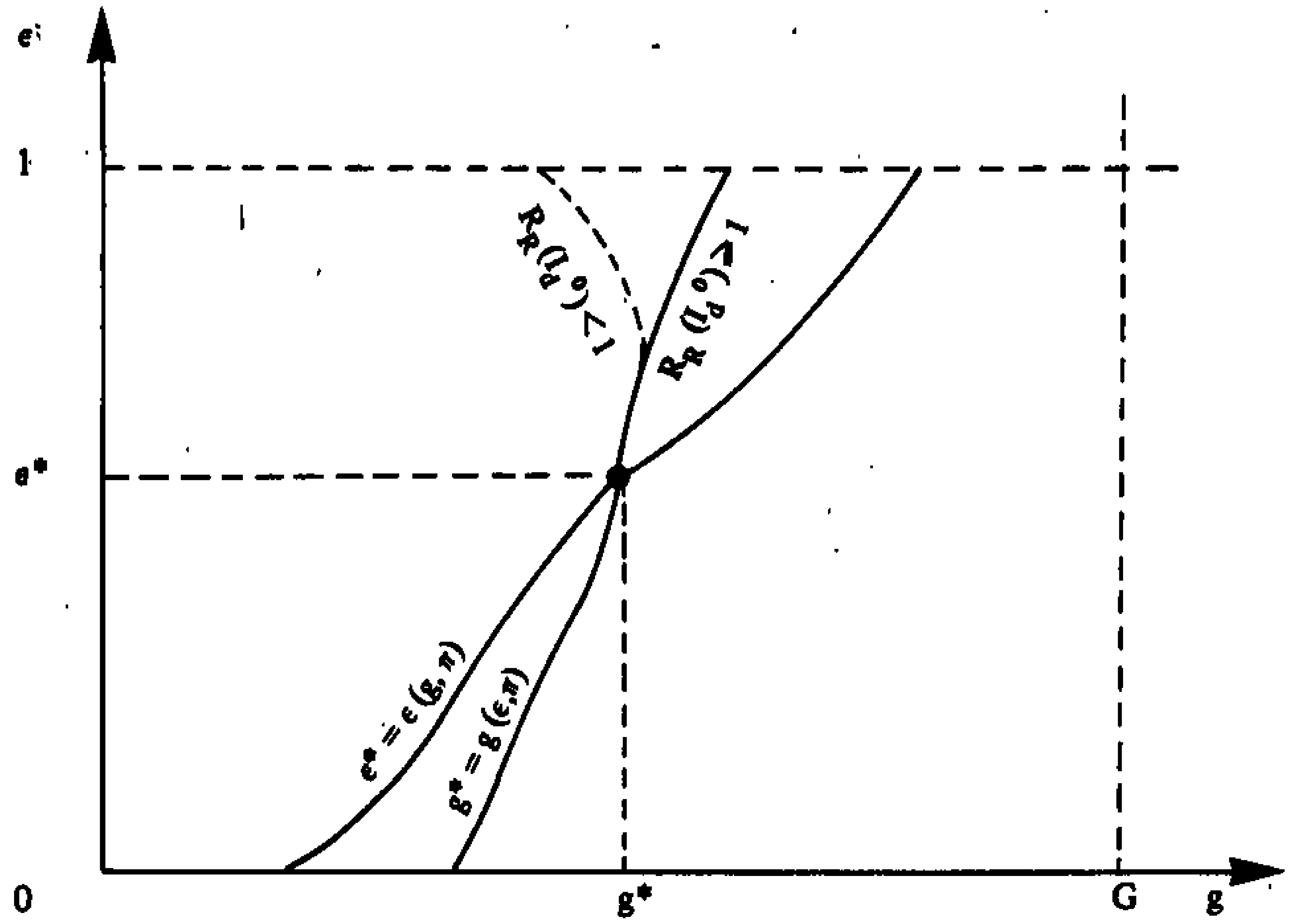


Figure 4: Determination of the individual's equilibrium of ϵ^* and g^* for increasing relative risk aversion.

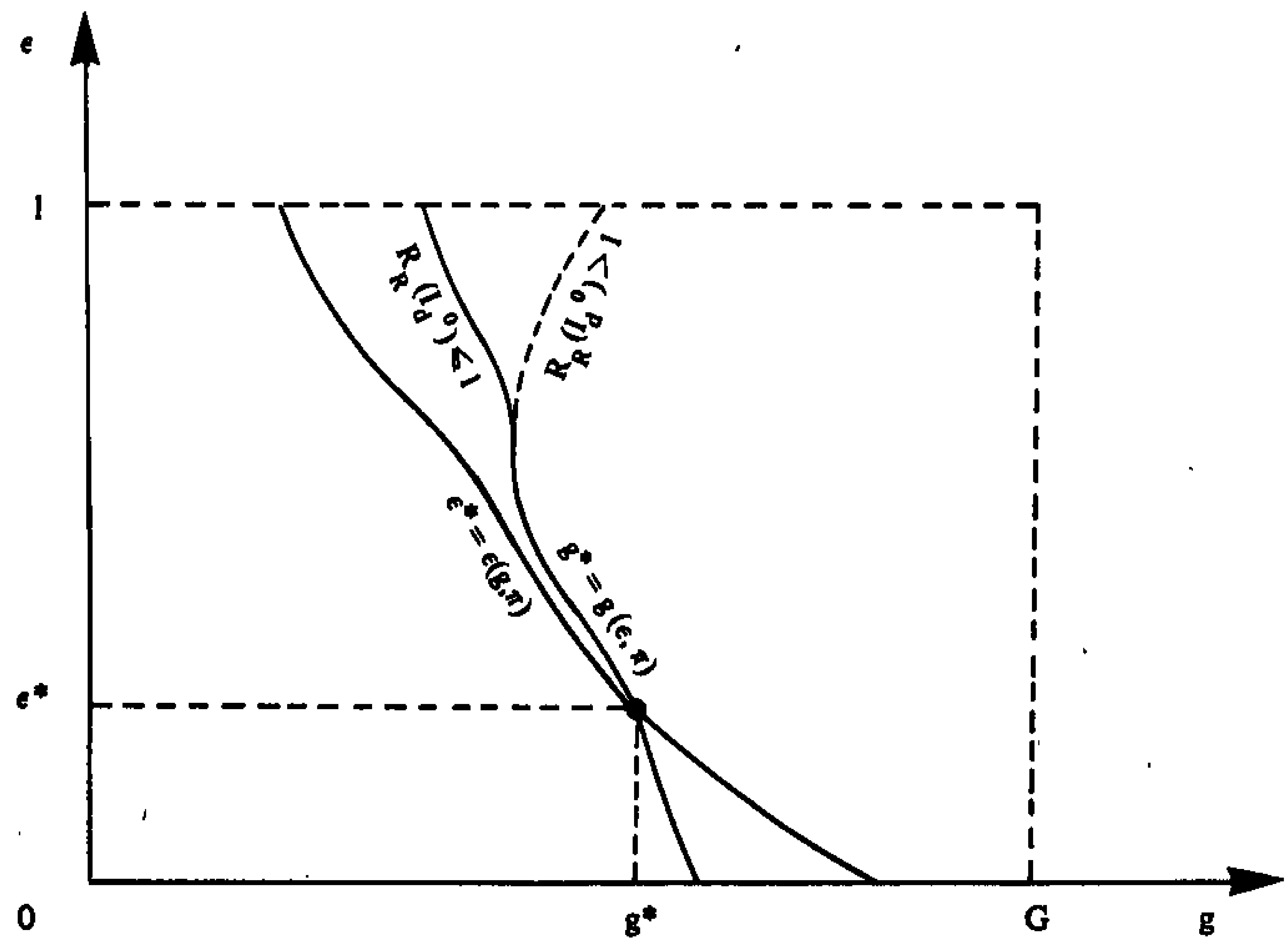


Figure 5: Determination of the individual's equilibrium of ϵ^* and g^* for decreasing relative risk aversion.

which is the second-order condition on both variables for the existence of an optimum.

(c) *Decreasing Relative Risk Aversion:* $R_R(I_d^{nc}) < R_R(I_d^{c})$

Under this assumption as well we should first identify the value of the relative risk aversion measure in the full report case. If the values of w and t are such that for $\epsilon = 1$ we have $R_R(I_d^{o}) > 1$, then $1 - R_R(I_d^{nc}) = 1 - R_R(I_d^{c}) < 0$, so that if tax evasion is profitable, (20) implies that $\frac{\partial g^*}{\partial \epsilon} > 0$. When ϵ is reduced, I_d^{c} decreases while I_d^{nc} increases so that $-(b-1)\,pU'(I_d^{c})\,[\,1 - R_R(I_d^{c})\,]$ is more positive while $(1-p)\,U'(I_d^{nc})\,[\,1 - R_R(I_d^{nc})\,]$ is less negative. For sufficiently small values of ϵ, $\frac{\partial g^*}{\partial \epsilon} < 0$.

By similar considerations it can be argued that if $R_R(I_d^{o}) \leq 1$, then $\frac{\partial g^*}{\partial \epsilon} < 0$ for each $\epsilon < 1$. Using (25) one is assured that under this assumption of decreasing relative risk aversion the individual's equili brium satisfies $\left.\frac{\partial g^*}{\partial \epsilon}\right|_{\epsilon = \epsilon^*} < 0$.

These findings, along with (19), are illustrated in Figure 5, where stability considerations are similar to those discussed in section (b).

III. THE SENSITIVITY OF INDIVIDUAL'S EQUILIBRIUM TO THE PARAMETERS OF THE MODEL: COMPARATIVE STATICS

Let us examine now how changes in the various parameters affect the simultaneous solutions of labor supply and the proportion of income declared. To do that we may differentiate (16) and (17) with respect to π to obtain, respectively,

$$\frac{\partial \epsilon^*}{\partial \pi} = \frac{\partial \epsilon^*}{\partial g}\,\frac{dg^*}{\partial \pi} + \frac{\partial \epsilon^*}{\partial \pi} \tag{26}$$

$$\frac{\partial g^*}{\partial \pi} = \frac{\partial g^*}{\partial \epsilon}\,\frac{d\epsilon^*}{d\pi} + \frac{\partial g^*}{\partial \pi} \tag{27}$$

Hence --

$$\frac{d\epsilon^*}{d\pi} = \frac{\dfrac{\partial \epsilon^*}{\partial g}\,\dfrac{\partial g^*}{\partial \pi} + \dfrac{\partial \epsilon^*}{\partial \pi}}{1 - \dfrac{\partial \epsilon^*}{\partial g}\,\dfrac{\partial g^*}{\partial \epsilon}} \tag{28}$$

$$\frac{dg^*}{d\pi} = \frac{\dfrac{\partial g^*}{\partial \epsilon}\,\dfrac{\partial \epsilon^*}{\partial \pi} + \dfrac{\partial g^*}{\partial \pi}}{1 - \dfrac{\partial \epsilon^*}{\partial g}\,\dfrac{\partial g^*}{\partial \epsilon}} \tag{29}$$

From the second-order conditions for the existence of an optional solution (and from Figures 3–5) it follows that the denominator of (28) and (29) is always positive. To determine the signs of (28) and (29) for a certain parameter, one should find first the partial sensitivity of each decision variable to changes in this parameter.

(a) A Change in the Market Wage Rate

Differentiating the optimum condition (11) with respect to w yields

$$\frac{\partial \epsilon^*}{\partial w} = \frac{tg(b-1)\,pU'(I_d{}^c)}{D}\,[\,R_R(I_d{}^c) - R_R(I_d{}^{nc})\,] \tag{30}$$

so that, similar to (19),

$$R_R(I_d^{nc}) \gtreqless R_R(I_d^{c}) <\!\!=\!\!> \frac{\partial \epsilon^*}{\partial w} \gtreqless 0 \tag{31}$$

That is, *an increase in the market wage rate for a given g will increase, leave constant, or* **decrease the optimal proportion of actual income declared if the relative risk aversion is an increasing, constant or decreasing function of disposable income.**

In the same way, differentiating the optimum condition (12) with respect to w yields

$$\frac{\partial g^*}{\partial w} = -\frac{1}{wB}\Big\{ V'(G-g)\,[\,1 - R_R(I_d^{nc})\,] + $$
$$+\, wpU'(I_d^{c})\,[\,1 - t(\epsilon + b\,(1-\epsilon))\,]\,[R_R(I_d^{nc}) - R_R(I_d^{c})]\Big\} \tag{32}$$

For $R_R(I_d^{nc}) = R_R(I_d^{c})$ the right hand term of (32) falls, so that

$$R_R(I_d) \gtreqless 1 <\!\!=\!\!> \frac{\partial g^*}{\partial w} \lesseqgtr 0 \tag{33}$$

While for $R_R(I_d^{nc}) \neq R_R(I_d^{c})$ the sign of (32) is unclear. For $R_R(I_d^{nc}) > R_R(I_d^{c})$ it is positive as long as $R_R(I_d^{nc}) \leq 1$ and may become negative for high values of w.

For $R_R(I_d^{nc}) < R_R(I_d^{c})$ *it is negative as long as* $R_R(I_d^{nc}) \geq 1$ *and may become positive* for high values of w.

(b) *A Change in the Income Tax Rate*

Differentiating (11) with respect to t yields

$$\frac{\partial \epsilon^*}{\partial t} = \frac{t\,(wg)^2(b-1)pU'(I_d^{c})}{D}\,[\,\epsilon R_A(I_d^{nc}) - (\epsilon + b\,(1-\epsilon))\,R_A(I_d^{c})\,] \tag{34}$$

where $R_A(I_d) = -\dfrac{U''(I_d)}{U'(I_d)}$ is the Arrow-Pratt absolute risk aversion measure.[7] It is thus clear from (34) that

$$\frac{R_A(I_d^{nc})}{R_A(I_d^{c})} \lesseqgtr \frac{\epsilon + b\,(1-\epsilon)}{\epsilon} <\!\!=\!\!> \frac{\partial \epsilon^*}{\partial t} \gtreqless 0 \tag{35}$$

Hence, for an absolute risk aversion which is a constant or decreasing function of income, $\frac{\partial \epsilon^*}{\partial t} > 0$, while no clear-cut result is achieved for an increasing absolute risk aversion.

As noted by Yitzhaki, the present assumption of a penalty function proportional to total taxes evaded cancels the negative substitution effect which a change in the tax rate has on declared income under the alternative assumption of a penalty function proportional to the level of unreported income. In this latter case an increase in the tax rate makes more expensive each reported unit of income, leading to a substitution effect towards underreporting, since the expected penalty per unit of income remains unchanged. However, under the present assumption, an increase in the tax rate makes more expensive as well the unreported unit of income, thus neutralizing this substitution effect. Nevertheless, we can still distinguish here between two different effects on the level of income declared. On the one hand, an increase in the tax rate reduces both I_d^{nc} and I_d^c for each ϵ, thus creating an income effect according to the nature of risk aversion; If absolute risk aversion is a decreasing or an increasing function of income, the reduction in disposable income will cause an increase or a decrease in income declared, respectively, while if absolute risk aversion is constant, the reduction in disposable income will not affect the level of the illegitimate activity. On the other hand, an increase in tax rate causes a greater reduction in I_d^c than in I_d^{nc}, thus creating a positive substitution effect towards an increase in income declared. Hence, if absolute risk aversion is a decreasing function of income, both effects pull in the same direction toward increasing income declared; If absolute risk aversion is independent of income, the second effect assures an increase in income declared; while if absolute risk aversion is an increasing function of income, the two effects pull in opposite directions.

Differentiating now (12) with respect to t yields

$$\frac{\partial g^*}{\partial t} = \frac{w}{B} \Big\{ \epsilon(1-p)\, U'(I_d^{nc}) [1 - R_R (I_d^{nc})] + + (\epsilon + b(1-\epsilon))\, pU'(I_d^c) [1 - R_R (I_d^c)] \Big\} \tag{36}$$

For $R_R(I_d^{nc}) = R_R(I_d^c)$ it is immediately apparent that

$$R_R(I_d) \gtrless 1 \Longleftrightarrow \frac{\partial g^*}{\partial t} \lessgtr 0 \tag{37}$$

[7] The sign of (34) depends upon the absolute and not the relative risk aversion, as in (31), for example. In this case income from work remains constant so that a change in t affects the proportion of income declared in the same way as it affects the absolute level of declared income.

while for $R_R(I_d^{nc}) \neq R_R(I_d^{c})$ the sign of (36) is not clear. For $R_R(I_d^{nc}) > R_R(I_d^{c})$ it is negative as long as $R_R(I_d^{nc}) \leq 1$ and may become positive for low values of t. For $R_R(I_d^{nc}) < R_R(I_d^{c})$ it is positive as long as $R_R(I_d^{nc}) \geq 1$ and may become negative for low values of t.

(c) A Change in the Penalty Rate

Differentiating (11) with respect to b yields

$$\frac{\partial \epsilon^*}{\partial b} = - \frac{twg\,(b-1)\,pU'(I_d^{c})}{D} \left[\frac{1}{b-1} + R_A(I_d^{c})(1-\epsilon)\,twg \right] > 0 \qquad (38)$$

so that, independent of the attitude towards risk, an increase in the penalty rate will always increase the proportion of actual income declared.

Similarly, differentiating (12) with respect to b yields

$$\frac{\partial g^*}{\partial b} = \frac{(1-\epsilon)\,twpU'(I_d^{c})}{B} \left[1 - R_R(I_d^{c}) \right] \qquad (39)$$

so that a constant relative risk aversion implies

$$R_R(I_d) \gtrless 1 \iff \frac{\partial g^*}{\partial b} \lessgtr 0 \qquad (40)$$

For relative risk aversion which varies with income the sign of (39) is positive as long as $R_R(I_d) > 1$. If relative risk aversion is an increasing function of income the sign may change for sufficiently high values of b, while if relative risk aversion is a decreasing function of income, we may expect a change in signs to take place for sufficiently small values of b.

(d) A Change in the Probability of Conviction

Differentiating (11) with respect to p yields

$$\frac{\partial \epsilon^*}{\partial p} = - \frac{twgU'(I_d^{nc})}{pD} > 0 \qquad (41)$$

so that, similar to a change in the penalty rate, an increase in the probability of conviction will always increase the proportion of actual income declared independent of the attitude toward risk.

Finally differentiating (12) with respect to p yields

$$\frac{\partial g^*}{\partial p} = \frac{1}{gB}[\,U'(I_d^{nc})\,I_d^{nc} - U'(I_d^{c})\,I_d^{c}\,] \tag{42}$$

so that

$$\frac{U'(I_d^{nc})\,I_d^{nc}}{U'(I_d^{c})\,I_d^{c}} \gtrless 1 \iff \frac{\partial g^*}{\partial p} \lesseqgtr 0 \tag{43}$$

Alternatively, defining $\eta_{U(I_d), I_d} = \frac{U'(I_d)\,I_d}{U(I_d)}$ as the utility elasticity of disposable income, one can argue that

$$\eta_{U(I_d^{nc}), I_d^{nc}} > \eta_{U(I_d^{c}), I_d^{c}} \implies \frac{\partial g^*}{\partial p} > 0 \tag{44}$$

That is, if the utility elasticity is a constant or an increasing function of income, an increase in the probability of conviction will reduce labor hours (for a given ϵ). On the other hand, if the utility elasticity is a decreasing function of income, the sign of (42) is not clear.

IV. THE ADJUSTMENT OF LABOR SUPPLY TO INCOME TAX EVASION

The results achieved in the previous chapter emphasized that the effect of changes in the parameters of the model on the optimal solutions of labor supply and income declared depends usually on the nature of risk aversion. After identifying the partial reaction of each decision variable to a change in each parameter, we are now ready to determine the full sensitivities of the optimal solutions for taxpayers whose attitude toward risk satisfies the more acceptable presumption of non-increasing absolute and non-decreasing relative risk aversion.

(a) Decreasing Absolute and Constant Relative Risk Aversion

Under this assumption, the individual's tendency to risk a given sum increases with income $[R_A(I_d^{nc}) < R_A(I_d^{c})]$, while his tendency to risk a given proportion of his income is independent of income $[R_R(I_d^{nc}) = R_R(I_d^{c})]$. The optimal declared proportion of actual income is thus independent of labor supply, and by a successive substitution of (19), (31), (35), (38), (41) into (28), it follows that

$$\frac{d\epsilon^*}{dw} = 0; \quad \frac{d\epsilon^*}{dt} > 0; \quad \frac{d\epsilon^*}{db} > 0; \quad \frac{d\epsilon^*}{dp} > 0 \tag{45}$$

That is, an increase in the market wage rate does not affect the optimal proportion of income declared, while an increase in one of the governmental parameters, such as the income tax rate, the penalty rate, or the probability of conviction, will tend to raise the proportion declared.

Examining now the effects on the optimal hours of work, it is necessary to distinguish between three different cases according to the absolute value of the relative risk aversion measure:

(1) $R_R(I_d) < 1$, which is satisfied by utility functions of the type $U = AI_d^{1-\alpha}$, where $A > 0$; $0 < \alpha < 1$; $R_R(I_d) = \alpha$.

(2) $R_R(I_d) = 1$, which is satisfied by utility functions of the type $U = A\ln I_d$, where $A > 0$.

(3) $R_R(I_d) > 1$, which is satisfied by utility functions of the type $U = A - BI_d^{-\alpha}$, where $\alpha, A, B > 0$; $R_R(I_d) = 1 + \alpha$.

Substituting successively (33), (37), (40), (43) into (29) and recalling that under the present assumption of constant relative risk aversion $\left.\frac{\partial g^*}{\partial \epsilon}\right|_{\epsilon = \epsilon^*} = 0$, it follows that

$$R_R(I_d) \lesseqgtr 1 \Longleftrightarrow \frac{dg^*}{dw} \gtreqless 0;\ \frac{dg^*}{dt} \lesseqgtr 0;\ \frac{dg^*}{db} \lesseqgtr 0;\ \frac{dg^*}{dp} \lesseqgtr 0 \tag{46}$$

Hence, an increase in the market wage rate will raise, leave constant or reduce time allocated to work, while an increase in one of the governmental parameters will reduce, leave constant, or raise it, accordingly as the relative risk aversion measure is less, equal, or greater than unity, respectively. Comparing (46) with (45) we may also conclude that when the relative risk aversion measure is constant, the optimal solutions of the model react in opposite directions to a change in any governmental parameter if the value of the measure is less than unity, but in the same direction if its value exceeds unity.

For an explicit derivation of the labor supply functions we may now choose the following utility functions defined on leisure and disposable income

$$U(I_d, G-g) = I_d^{1/2} + (G-g)^{1/2} \qquad \text{for } R_R(I_d) < 1 \tag{47}$$

$$U(I_d, G-g) = \ln I_d + \ln(G-g) \qquad \text{for } R_R(I_d) = 1 \tag{48}$$

$$U(I_d, G-g) = U_0 - \frac{1}{I_d} - \frac{1}{G-g} \qquad \text{for } R_R(I_d) > 1 \tag{49}$$

Expressing successively the optimum condition (11) in terms of (47), (48) and (49), the optimal proportion of actual income declared $[\text{ for } 1 < b < \frac{1}{p}\ ]$ can be given by

$$\epsilon^* = \frac{[\,p(b-1)\,]^i + (tb-1)(1-p)^i}{\{t\ \ [\,p(b-1)\,]^i + (b-1)(1-p)^i\}} \tag{50}$$

where $i = 2, 1, ½$ for $R_R(I_d) < 1$, $R_R(I_d) = 1$, $R_R(I_d) > 1$, respectively. By an appropriate substitution of (50) into the optimum condition (12), we obtain the labor supply functions adjusted to tax evasion as (See Math. App. A).

$$g^* = G\,\frac{\phi(w,t,b,p)}{1+\phi(w,t,b,p)}, \qquad \text{For } R_R(I_d) < 1 \tag{51}$$

where $\phi(w,t,b,p) = (1-t)\,w\,\dfrac{b\,[\,p^2(b-1) + (1-p)^2\,]}{b-1}$ is the "cash equivalent" of the uncertain disposable wage which satisfies $U(\phi g) = EU(I_d)$ for (47).

$$g^* = \frac{G}{2}, \qquad \text{For } R_R(I_d) = 1 \tag{52}$$

$$g^* = G \frac{1}{1 + [\psi(w, t, b, p)]^{1/2}}, \qquad \text{For } R_R(I_d) > 1 \tag{53}$$

where $\psi(w, t, b, p) = (1 - t)\, w \dfrac{b}{[p^{1/2} + (b-1)^{1/2}(1-p)^{1/2}]^2}$ is the "cash equivalent" of the the uncertain disposable wage which satisfies $U(\psi g) = EU(I_d)$ for (49).

Hence, in a tax evasion economy, an adequate "cash equivalent" concept can be introduced for each utility function to replace the legal disposable wage as an explanatory variable of labor supply.[8] It is apparent from (51) and (53) that for $R_R(I_d) < 1$ the labor supply is an increasing function of $\phi(w, t, b, p)$, while for $R_R(I_d) > 1$ it is a decreasing function of $\psi(w, t, b, p)$. Moreover, (51) and (53) enable us to compare labor supply in the full report case to that of tax evasion for each w and t: In the full-report case the disposable wage ceases to be a random variable and is given by $(1 - t)\, w$. It is easy to see that both $\phi(w, t, b, p)$ and $\psi(w, t, b, p)$ are greater than $(1 - t)\, w$, since they are each a product of the legal disposable wage and a term which exceeds unity for b and p satisfying $bp < 1$. We thus conclude that for $R_R(I_d) < 1$, tax evasion is accompanied by an increased labor supply, while for $R_R(I_d) > 1$ it is accompanied by a decreased supply, for each w and t.

These conclusions can also be reached by diagramatical examination of the changes in the individual's equilibrium resulting from a change in the market wage rate or the income tax rate: An increase in w for a given t does not affect ϵ^*, but causes $g(\epsilon, \pi_w, w)$[9] to shift from left to right for $R_R(I_d) < 1$ (Figure 6) or from right to left for $R_R(I_d) > 1$ (Figure 8).[10] The allocations of time to work for the full-report case are indicated by the points A, B, C along the dotted line of $\epsilon = 1$, while for each $\epsilon^* < 1$ they are obtained by the points A′, B′, C′, indicating more hours of work for each $w > 0$ in the case of $R_R(I_d) < 1$, and less hours of work in the case of $R_R(I_d) > 1$. Similarly, an increase in t for a given w causes $g(\epsilon, \pi_t, t)$ to shift from right to left for $R_R(I_d) < 1$ (Figure 7) or from left to right for $R_R(I_d) > 1$ (Figure 9), raising its turning point along with ϵ^*. Comparing points A, B, C with A′, B′, C′ it is apparent that for each $t < 1$, the individual allocates more hours to work when evading taxes than when

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8 This does not include the case of $R_R(I_d) = 1$ where the individual always allocates half his time to work. The decisions on labor supply and tax evasion are thus entirely independent.

9 π_i will be hereafter used to denote the vector of the given parameters excluding parameter i.

10 For $w = 0$ it follows from (21) that $\dfrac{\partial g^*}{\partial w} = 0$, so that $g(\epsilon, \pi_w, o)$ interlinks with the vertical zero axis for $R_R(I_d) < 1$ and with the vertical G axis for $R_R(I_d) > 1$.

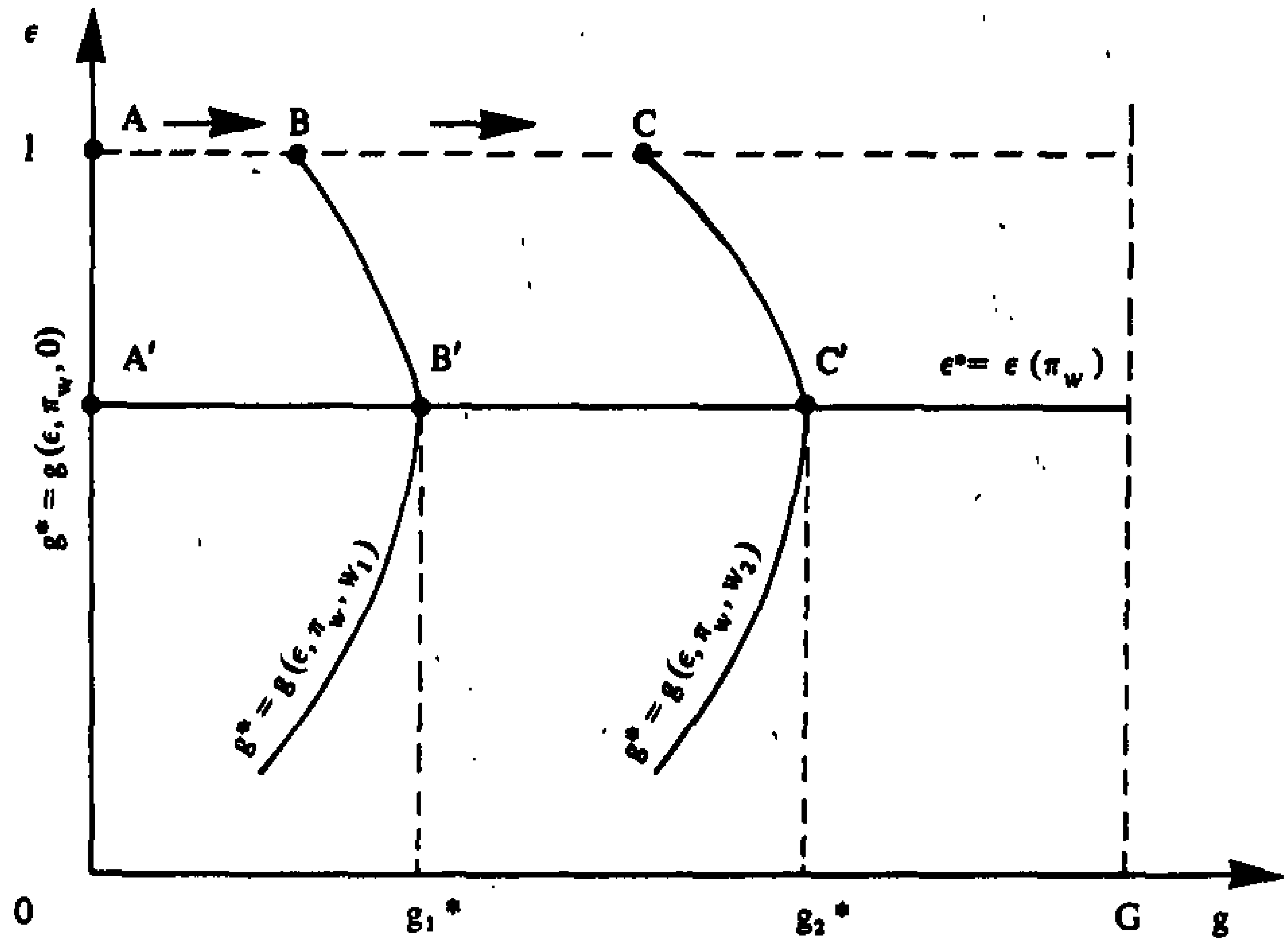

Figure 6: Changes in the individual's equilibrium resulting from an increase in w for $R_R(I_d) < 1$.

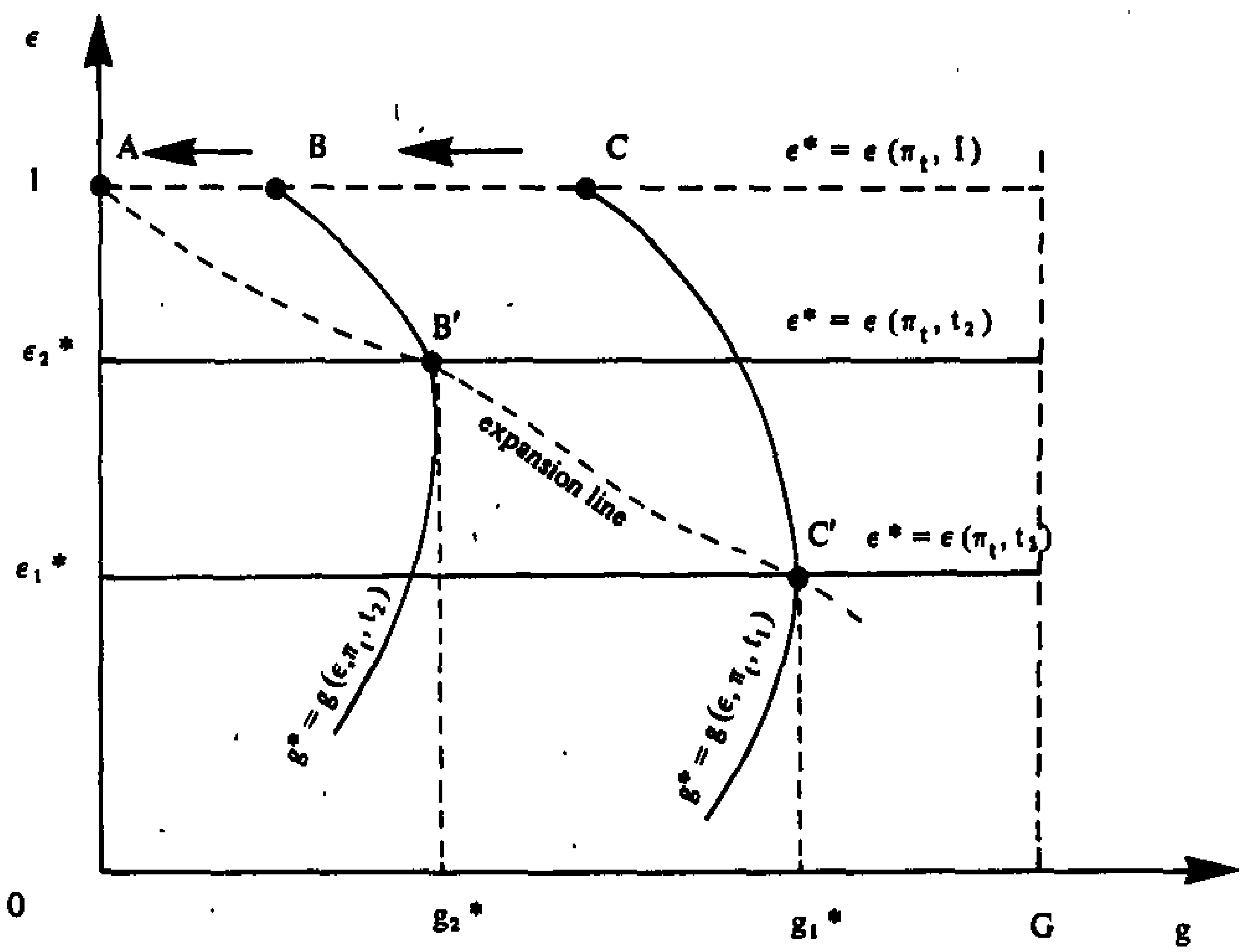

Figure 7: Changes in the individual's equilibrium resulting from an increase in t for $R_R(I_d) < 1$.

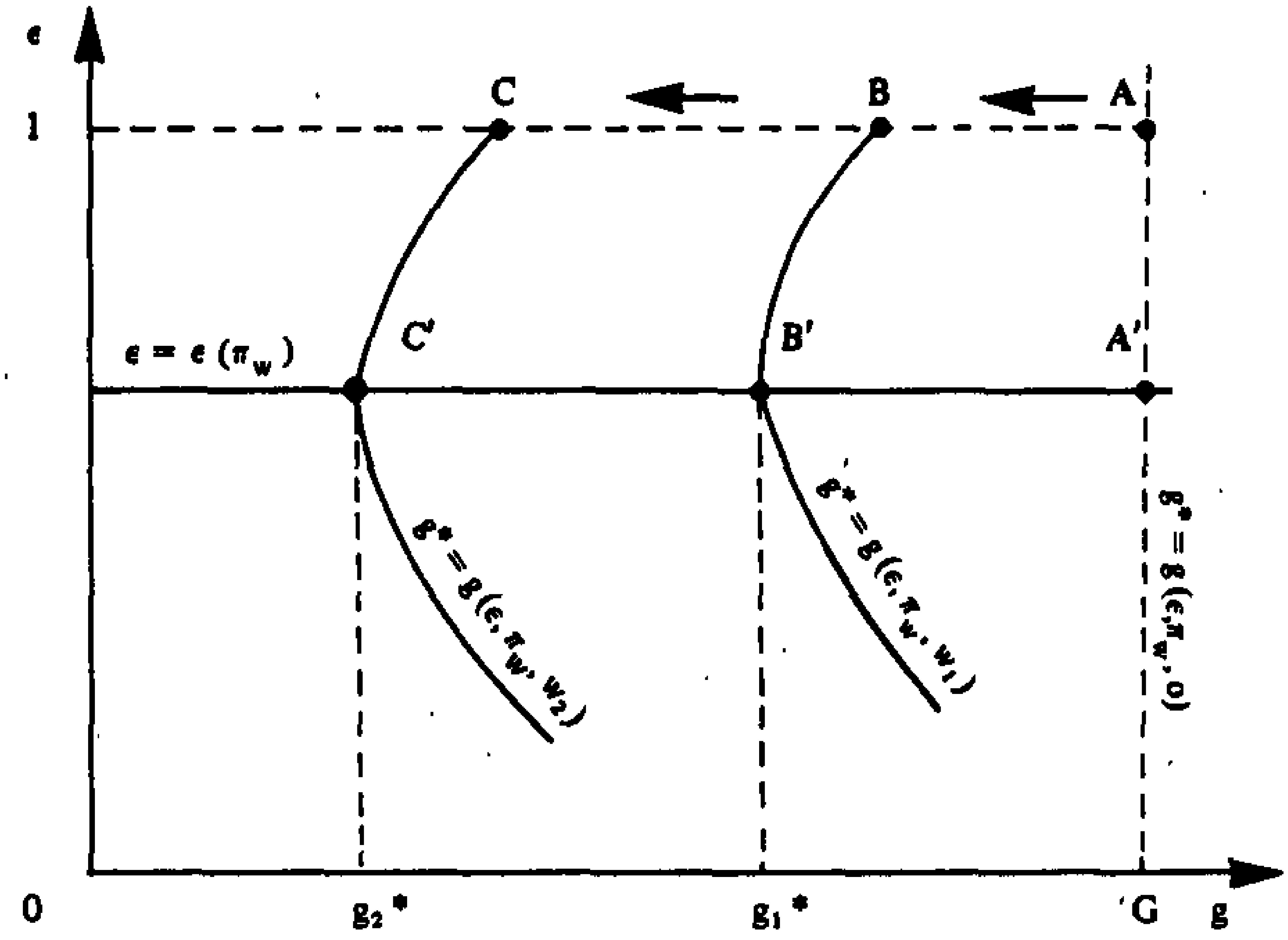

Figure 8: Changes in the individual's equilibrium resulting from an increase in w for $R_R(I_d) > 1$.

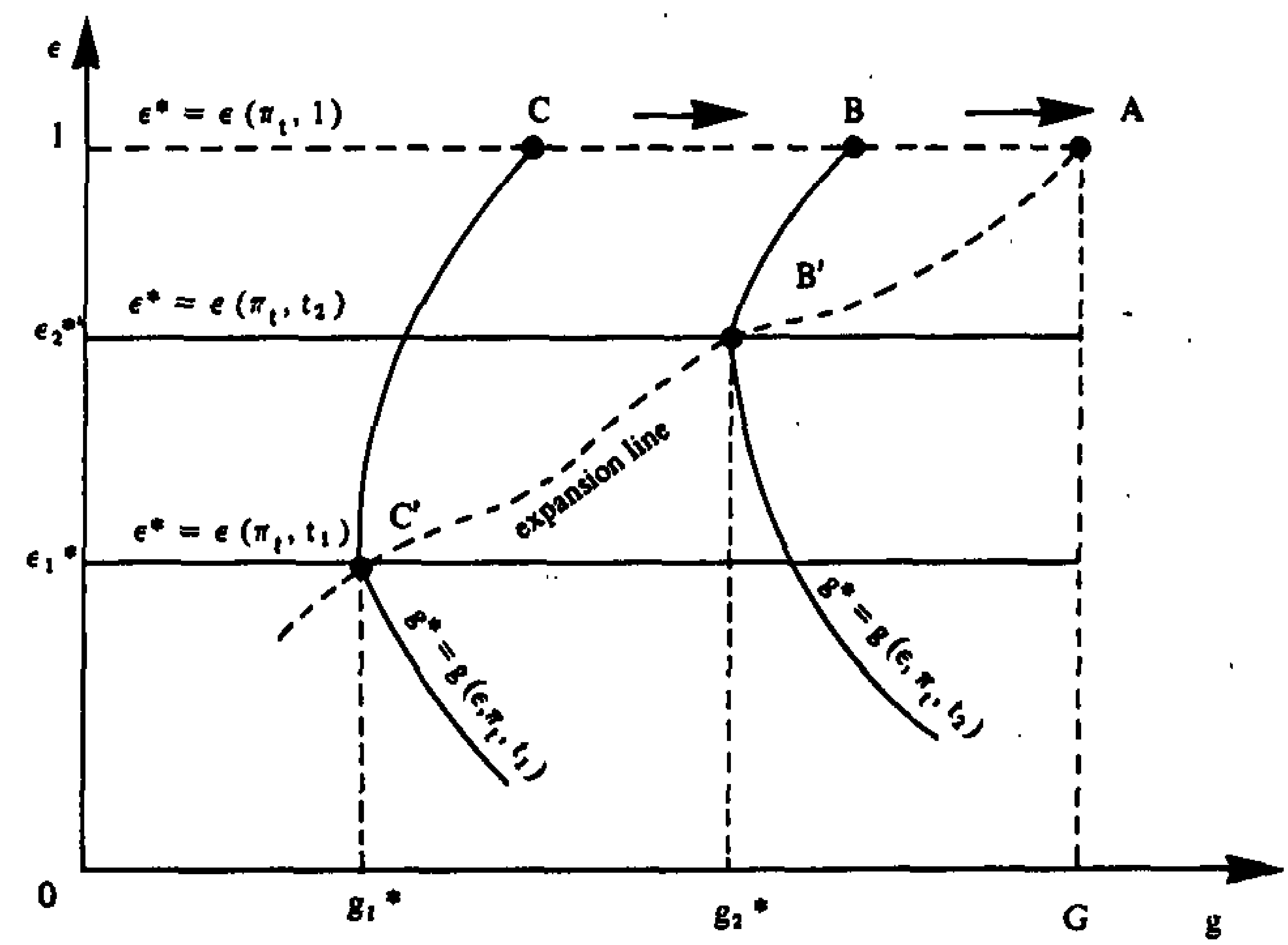

Figure 9: Changes in the individual's equilibrium resulting from an increase in t for $R_R(I_d) > 1$.

reporting his true income in the case of $R_R(I_d) < 1$, and less hours to work in the case of $R_R(I_d) > 1$. These results enable us to draw full-report labor supply curves along with labor supply curves of tax evasion as a function of w (Figures 10 and 12) and t (Figures 11 and 13).[11] Since $\frac{dg^*}{db}$; $\frac{dg^*}{dp}$ are both negative for $R_R(I_d) < 1$ and positive for $R_R(I_d) > 1$, it follows for both cases that the departure of each dotted tax evasion curve from its "original" full-report curve will be smaller the higher the value of the penalty rate or the probability of conviction.

(b) Constant Absolute and Increasing Relative Risk Aversion

Under this assumption the individual's tendency to risk a given sum is independent of his income $[\, R_A(I_d^{nc}) = R_A(I_d^{c})\,]$, while his tendency to risk a given proportion of income decreases as income rises. Utility functions which satisfy this assumption are exponential functions of the type $U = A - Be^{-\alpha I_d}$ where $\alpha, A, B > 0$; $R_A(I_d) = \alpha^2 B$.

In contrast to the previous assumption of constant relative risk aversion, the optimal proportion of actual income declared depends in this case on labor supply as well. Although the partial derivatives of ϵ^* with respect to any of the parameters $\left(\frac{\partial \epsilon^*}{\partial \pi}\right)$ are positive under the present assumption, the full sensitivity of ϵ^* to a change in one of the parameters $\left(\frac{d\epsilon^*}{d\pi}\right)$ is still unobtainable from (28), since the signs of the partial derivatives of g^* with respect to those parameters $\left(\frac{\partial g^*}{\partial \pi}\right)$ are not clear cut. For the same reason it is not possible to read from (29) the full reaction of g^* to a change in the parameters of the model $\left(\frac{dg^*}{d\pi}\right)$. We therefore derive explicitly the labor supply function adjusted to tax evasion, by choosing the following utility function defined on leisure and disposable income

$$U(I_d, G - g) = U_0 - e^{-I_d} - e^{-(G - g)} \qquad (54)$$

Expressing the optimum condition (11) in terms of (54), the optimal proportion of actual income declared, which is not independent of g, can be written as

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[11] To each value of the income tax rate in Figures 10 and 12 corresponds a different supply curve which lies more to the right as t is lower for $R_R(I_d) < 1$, or as t is higher for $R_R(I_d) > 1$. Similarly, for each value of the market wage rate in Figures 11 and 13 one may define a different supply curve which lies more to the right as w is higher for $R_R(I_d) < 1$, or as w is lower for $R_R(I_d) > 1$.

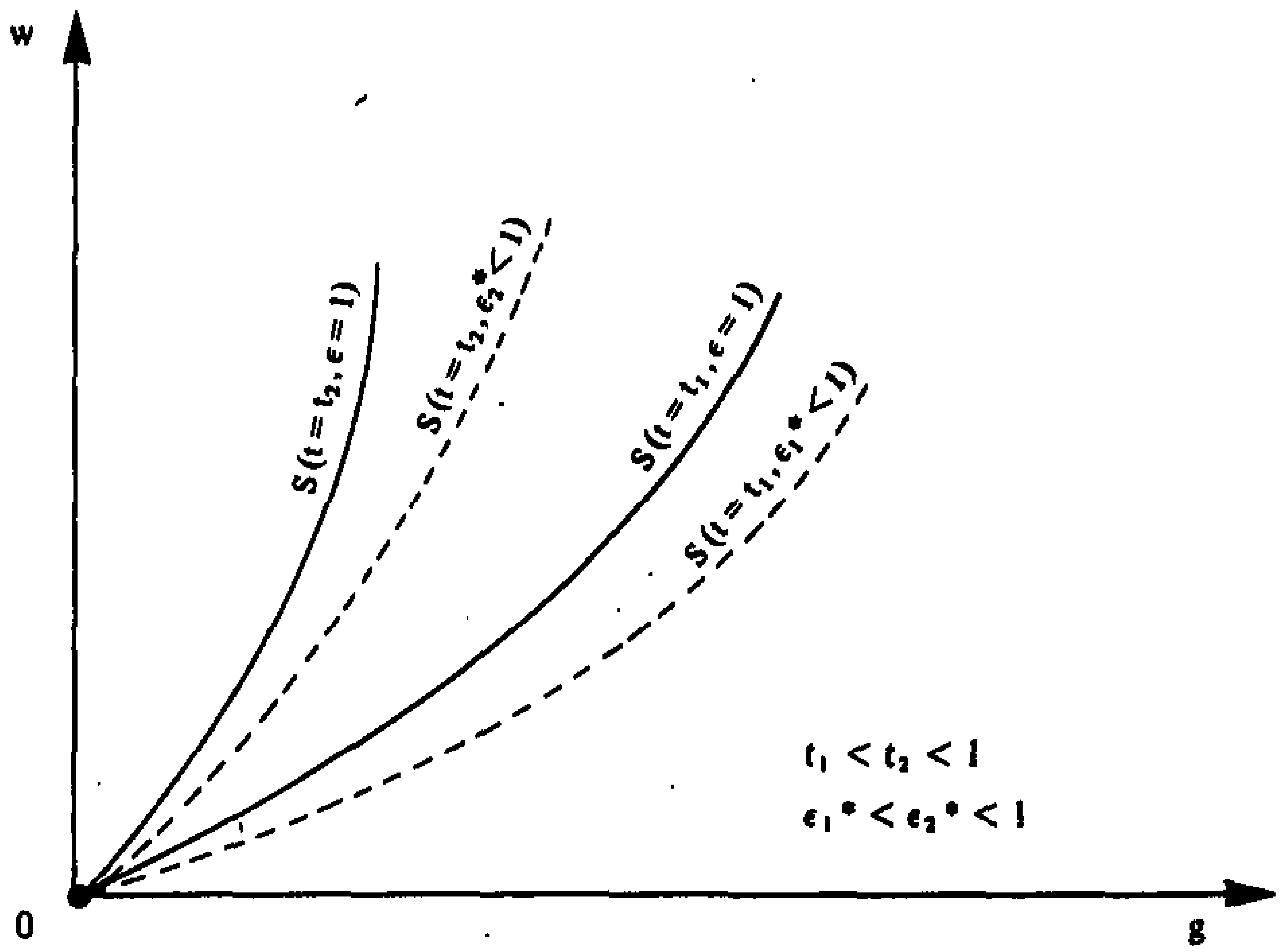

Figure 10: Labor supply as a function of the market wage rate for $R_R(I_d) < 1$: Full report ($\epsilon = 1$) and optimal tax evasion ($\epsilon^* < 1$).

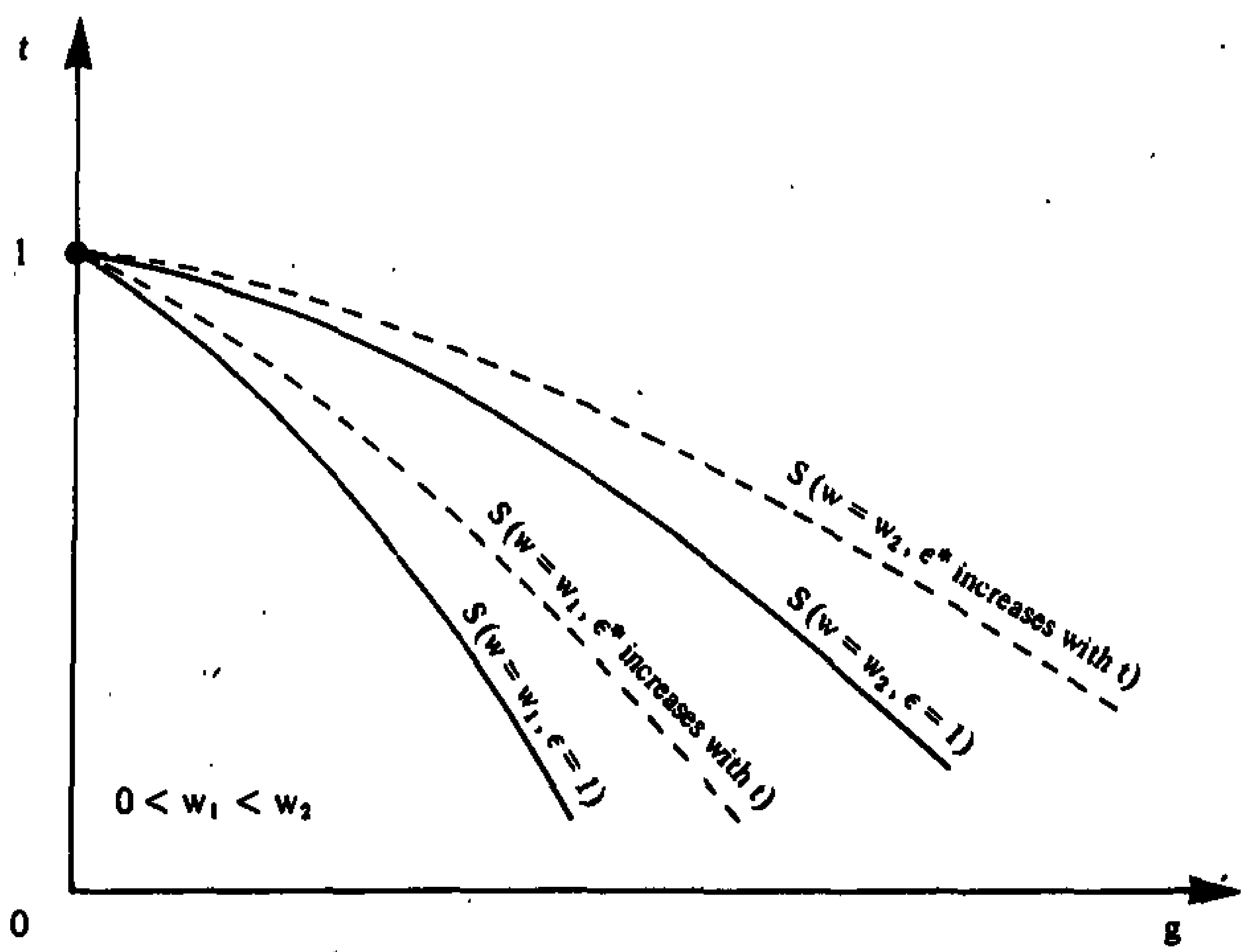

Figure 11: Labor supply as a function of the income tax rate for $R_R(I_d) < 1$: Full report ($\epsilon = 1$) and optimal tax evasion ($\epsilon^* < 1$).

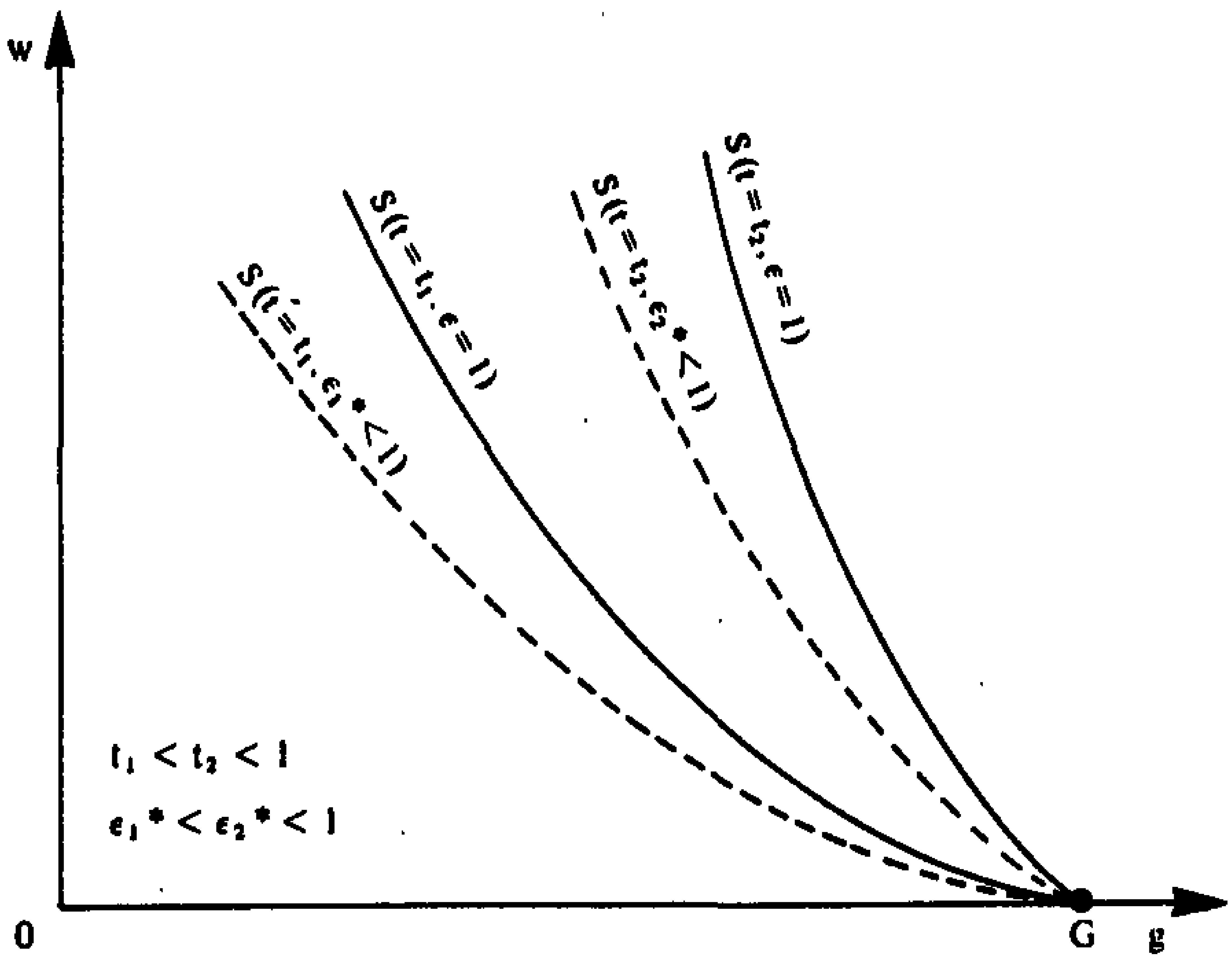

Figure 12: Labor supply as a function of the market wage rate for $R_R(I_d) > 1$: Full report ($\epsilon = 1$) and optimal tax evasion ($\epsilon^* < 1$).

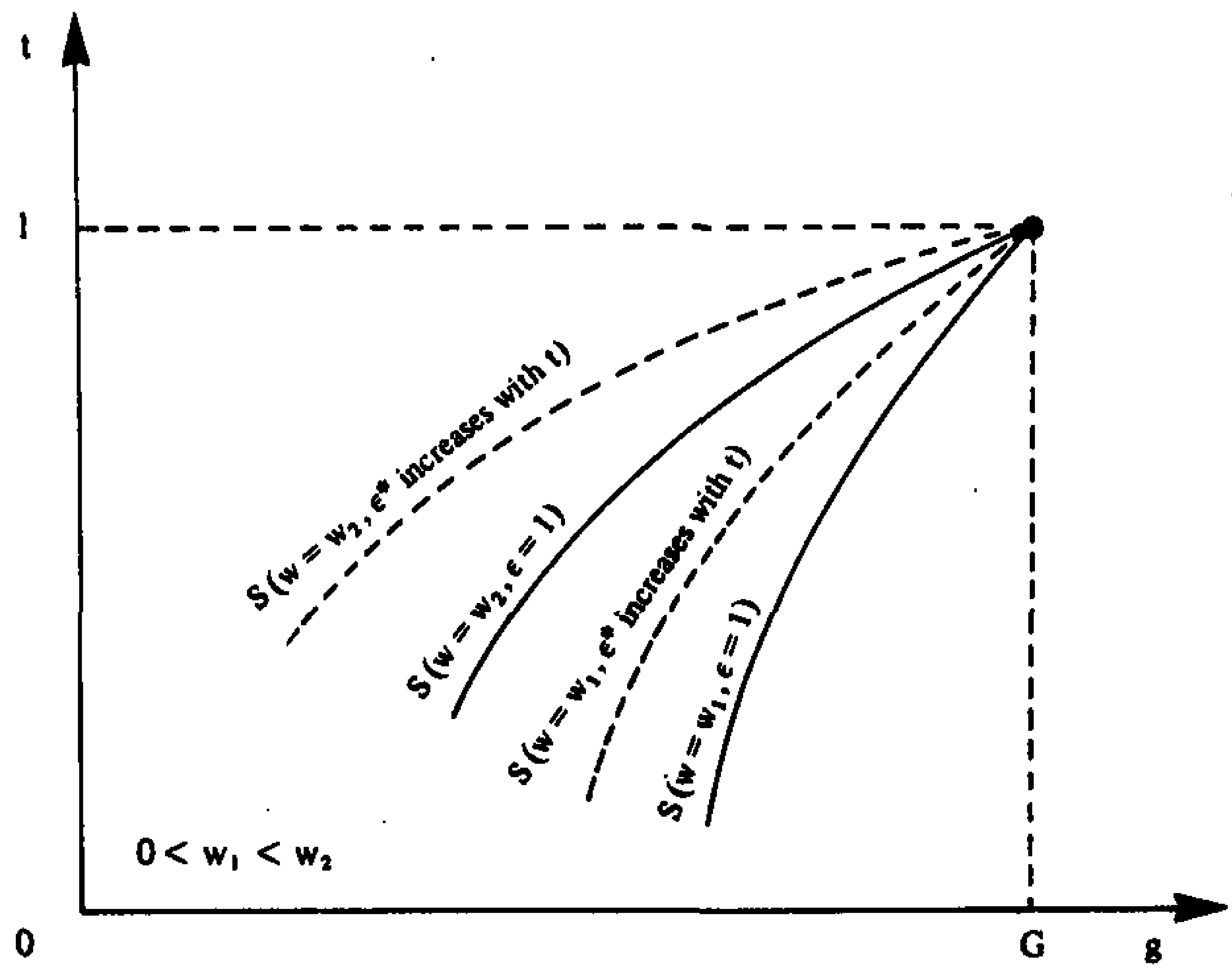

Figure 13: Labor supply as a function of the income tax rate for $R_R(I_d) > 1$: Full report ($\epsilon = 1$) and optimal tax evasion ($\epsilon^* < 1$).

$$\epsilon^* = 1 - \frac{1}{btwg} \ln \frac{1-p}{p(b-1)} \qquad (55)$$

while a substitution of (55) into the optimum condition (12) yields the following implicit labor supply function (See Math. App. A.)

$$g^* = \frac{G + \ln(1-t)w}{1 + \lambda(g^*, w, t, b, p)} \qquad (56)$$

where $\lambda(g, w, t, b, p) = (1-t)w + \frac{1}{g}\left[\frac{1}{b}\ln\frac{1-p}{p(b-1)} - \ln\frac{(1-p)b}{b-1}\right]$ is the "cash equivalent" of the uncertain disposable wage which satisfies $U[\lambda(g)g] = EU(I_d)$, and which obviously depends upon g as well. It is now obvious that for b and p satisfying $bp = 1$, $\lambda = (1-t)w$, since the multiplier of $\frac{1}{g}$ equals zero. However, since differentiating this multiplier with respect to b yields $-\frac{1}{b^2}\ln\frac{1-p}{p(b-1)} < 0$ and with respect to p yields $\frac{bp-1}{bp(1-p)} < 0$, it follows that for smaller b or p satisfying $bp < 1$, the multiplier of $\frac{1}{g}$ will be positive so that $\lambda(g, w, t, b, p) > (1-t)w$. We thus conclude from (56) that under this assumption of constant absolute and increasing relative risk aversion, tax evasion is accompanied by a decreased labor supply for each w and t.

To express g^* in terms of the parameters only, we can substitute $\lambda'(g, w, t, b, p)$ into (56) and obtain

$$g^* = \frac{G + \ln(1-t)w + \ln\frac{b(1-p)}{b-1} - \frac{1}{b}\ln\frac{1-p}{p(b-1)}}{1 + (1-t)w} \qquad (57)$$

Substituting now (57) into (55) enables us to obtain also the optimal proportion of actual income declared in terms of the parameters

$$\epsilon^* = \frac{G + \ln(1-t)w + \ln\frac{b(1-p)}{b-1} - \frac{1+w}{btw}\ln\frac{1-p}{p(b-1)}}{G + \ln(1-t)w + \ln\frac{b(1-p)}{b-1} - \frac{1}{b}\ln\frac{1-p}{p(b-1)}} \qquad (58)$$

By a differentiation of (57) and (58) with respect to w, t, b and p, it will be possible now to determine directly the full reaction of the two optimal solutions to changes in the parameters of the model:

Differentiating (57) with respect to w and t yields respectively

$$\frac{dg^*}{dw} = \frac{\frac{1}{w} - (1-t)\,g^*}{1 + (1-t)w} \tag{59}$$

$$\frac{dg^*}{dt} = \frac{-\frac{1}{1-t} + wg^*}{1 + (1-t)w} \tag{60}$$

so that

$$R_R(I_d) = (1-t)\,wg^* \lesseqgtr 1 \iff \frac{dg^*}{dw} \gtreqless 0;\ \frac{dg^*}{dt} \lesseqgtr 0 \tag{61}$$

While differentiating (58) with respect to w and t yields, respectively [where $\Delta > 0$ denotes the denominator of (58)]

$$\frac{d\epsilon^*}{dw} = \frac{1}{\Delta w}\left[\,1 - \epsilon^* + \frac{1}{btw}\ln\frac{1-p}{p\,(b-1)}\right] > 0 \tag{62}$$

$$\frac{d\epsilon^*}{dt} = \frac{1}{\Delta}\left[\,-\frac{1-\epsilon^*}{1-t} + \frac{1+w}{bwt^2}\ln\frac{1-p}{p\,(b-1)}\right] \tag{63}$$

so that by substituting ϵ^* of (57) into (63)

$$R_R(I_d) = (1-t)\,wg^* \lesseqgtr \frac{wt}{1+w} \iff \frac{d\epsilon^*}{dt} \lesseqgtr 0 \tag{64}$$

Considering (62) and (63) and recalling (26), it can be noted that the negative value of $\frac{dg^*}{dw}$ in the domain of $R_R(I_d) > 1$ is not sufficiently large to cause the negativity of $\frac{d\epsilon^*}{dw}$, while the negative value of $\frac{dg^*}{dt}$ in the domain of $R_R(I_d) < 1$ causes eventually the negativity of $\frac{d\epsilon^*}{dt}$ as well – for high enough values of t satisfying $R_R(I_d) < \frac{wt}{1+w}$.

Continuing to differentiate (57) and (58) with respect to b and p we obtain, respectively

$$\frac{dg^*}{db} = \frac{\ln \frac{1-p}{p(b-1)}}{b^2 [1 + (1-t)w]} > 0 \tag{65}$$

$$\frac{dg^*}{dp} = \frac{1 - bp}{bp(1-p)[1 + (1-t)w]} > 0 \tag{66}$$

$$\frac{d\epsilon^*}{db} = \frac{1}{\Delta}\left[\frac{1}{b(b-1)}\left(\frac{1+w}{tw} - 1\right) + \frac{1}{b^2}\left(\frac{1+w}{tw} - \epsilon^*\right) \ln \frac{1-p}{p(b-1)} > 0\right] \tag{67}$$

$$\frac{d\epsilon^*}{dp} = \frac{1}{\Delta bp(1-p)}\left[\frac{1+w}{tw} - bp(1-\epsilon^*) - \epsilon^*\right] > 0 \tag{68}$$

Hence, under the present assumption of constant absolute and increasing relative risk aversion, an increase in the penalty rate or the probability of conviction will raise both labor supply and the declared proportion of actual income

Using (56) and (61) we can demonstrate in Figures 14 and 15 the full-report labor supply curve along with the tax evasion curve, as a function of w and t respectively. Note that in comparison to the full-report labor supply, the tax evasion labor supply reaches its turning point at a higher w for each t (Figure 14), or at a lower t for each w (Figure 15). This occurs since for each w and t satisfying $(1-t)wg = 1$ in the full-report case, $(1-t)wg < 1$ when taxes are evaded. Hence the supply curve for a given t still rises, while the supply curve for a given w still descends. Finally, it is seen again by (65) and (66) that similar to the previous cases, the departure of each dotted tax evasion curve from its "original" full-report curve will be smaller the higher the value of the penalty rate or the probability of conviction.

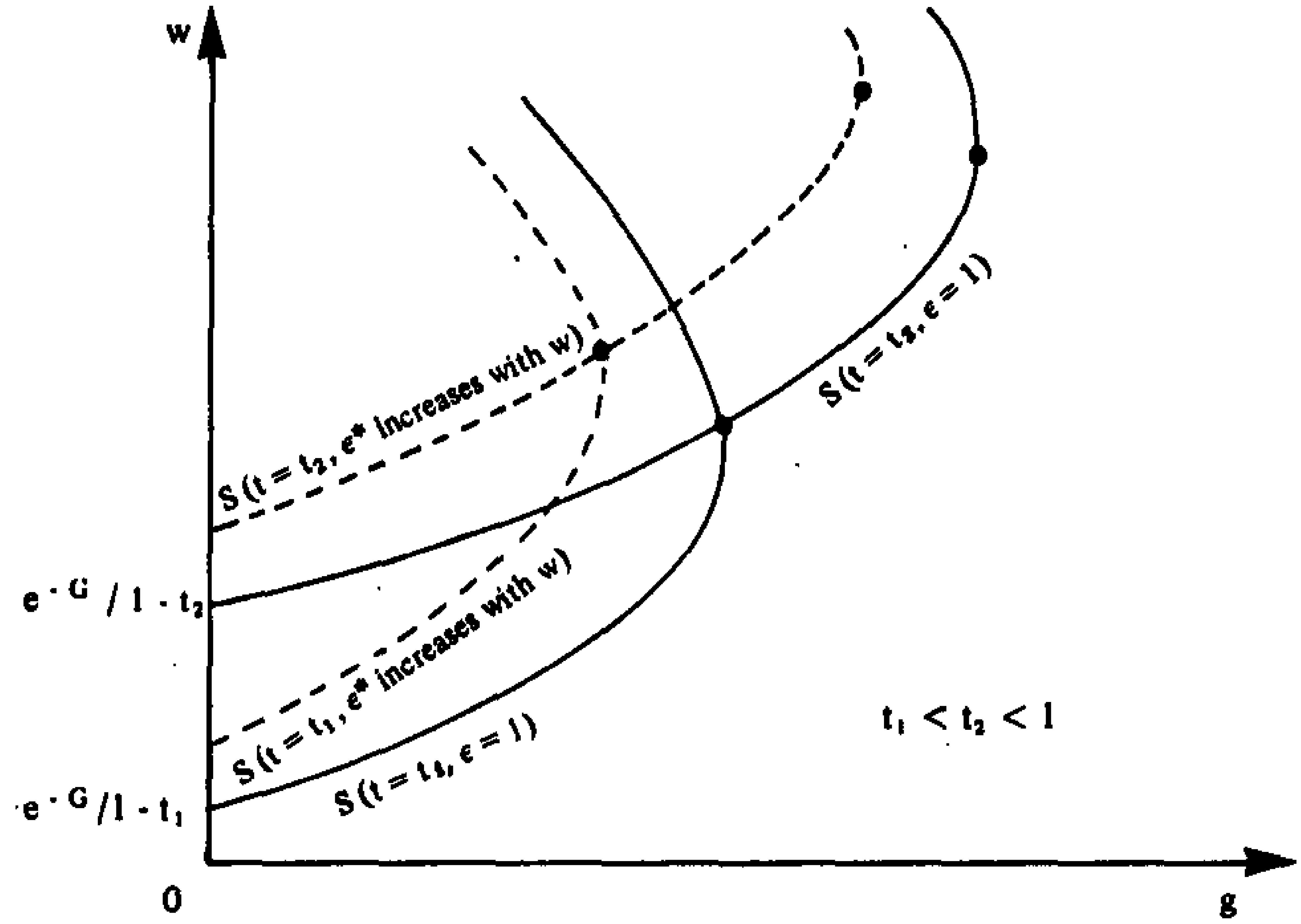

Figure 14: Labor supply as a function of the market wage rate for an increasing $R_R(I_d)$: Full report ($\epsilon = 1$) and optimal tax evasion ($\epsilon^* < 1$).

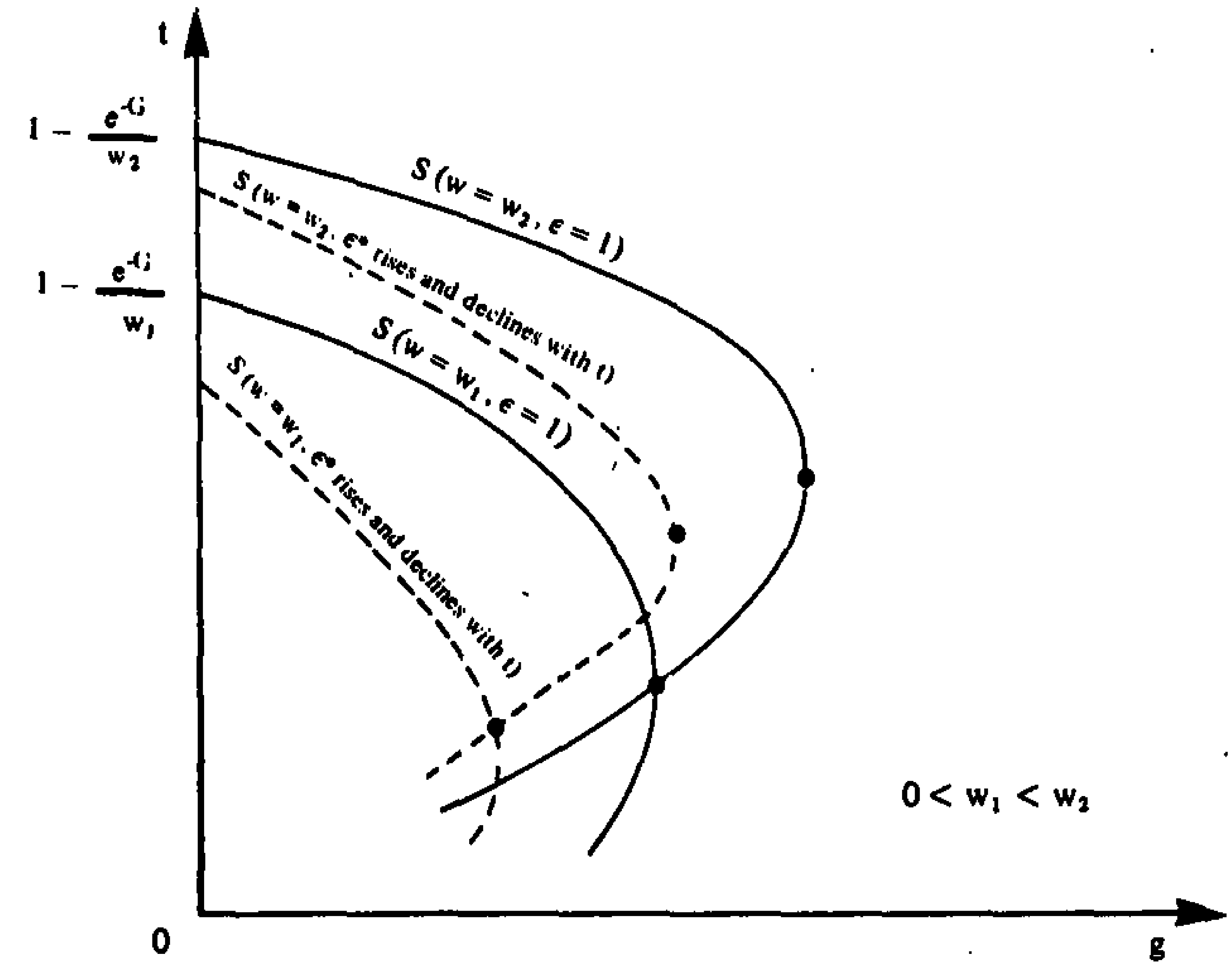

Figure 15: Labor supply as a function of the income tax rate for an increasing $R_R(I_d)$: Full report ($\epsilon = 1$) and optimal tax evasion ($\epsilon^* < 1$).

V. GENERAL EQUILIBRIUM: COMPARING BETWEEN ECONOMIES

Let us consider now two economies, A and B, in both of which there are N taxpayers having identical tastes and facing the same market wage rate per hour. Let us further assume that there is no tax evasion in economy A, while an optimal level of evasion takes place in Economy B (hence $bp < 1$ is satisfied). Suppose now that the government's expected receipts from taxes and fines in economy B is equal to the government's certain receipts from taxes in economy A. Does the total level of work in economy B exceed or fall short of that taking place in economy A?

To answer this question we first express the government's receipts from taxes in economy A as

$$T_A = Nt_A wg_A \quad . \tag{69}$$

and the government's expected receipts from taxes and fines in economy B as

$$E(T_B) = (1-p)N\epsilon^* t_B wg_B + pN[\epsilon^* + b(1-\epsilon^*)]t_B wg_B = \\ = Nt_B wg_B[\epsilon^*(1-bp)+bp] \tag{70}$$

Hence, (70) implies alternatively the conviction of pN taxpayers and the non-conviction of $(1-p)N$.

Let us examine first (69) and (70) for $t_A = t_B$. It is easily seen that $\epsilon^*(1-bp)+bp < 1$, so that if, for the same income tax rate, one also has $g_B \leqslant g_A$ it is obvious that $E(T_B) < T_A$. Restricting the discussion to utility functions which satisfy constant relative risk aversion, it follows from the previous chapter that the condition above holds for $R_R(I_d) \geqslant 1$. Since for these cases we also have $\frac{d\epsilon^*}{dt} > 0$; $\frac{dg^*}{dt} \geqslant 0$, it is clear that an equality of $E(T_B) = T_A$ requires $t_B > t_A$. In which economy then do we expect to find more hours allocated to work? For $R_R(I_d) = 1$ we trivially have $g_A = g_B = \frac{G}{2}$. For $R_R(I_d) > 1$ there is no clear-cut answer at this stage, since for $t_A \doteq t_B$ we have indeed $g_B < g_A$, but the higher value of t_B required to satisfy $E(T_B) = T_A$ raises g_B as well. For $R_R(I_d) < 1$ the problem is even more complicated since in this case we have $g_B > g_A$ for $t_A = t_B$, so that the relation between $E(T_B)$ and T_A for the same income tax rate is unclear from the beginning, and we are not able to determine whether t_B required for $E(T_B) = T_A$ is still greater than t_A.

Attempting a more rigorous approach, we have by the assumption of equal receipts in both economies

$$t_A g_A = [\,\epsilon^*(1 - bp) + bp\,]\, t_B g_B \tag{71}$$

Using (50) we may rewrite ϵ^* for the constant relative risk aversion case as

$$\epsilon^* = \frac{p^i (b-1)^i - (1-p)^i}{t_B\,[\,p^i (b-1)^i + (b-1)(1-p)^i\,]} + \frac{b\,(1-p)^i}{p^i (b-1)^i + (b-1)(1-p)^i} \tag{72}$$

and substituting (72) into (71)

$$t_A g_A = \left\{ \frac{p^i (b-1)^i - (1-p)^i}{p^i (b-1)^i + (b-1)(1-p)^i}(1 - bp) + \right.$$

$$\left. + \; t_B \left[\frac{b\,(1-p)^i}{p^i (b-1)^i + (b-1)(1-p)^i}(1 - bp) + bp \right] \right\} g_B =$$

$$= (\mu + t_B \delta)\, g_B \tag{73}$$

where $\mu + \delta = 1$ are functions of b and p only.[12] We thus have the following relation for t_B which satisfies the initial condition

$$t_B = \frac{1}{\delta g_B}(t_A g_A - \mu g_B) \tag{74}$$

Let us examine first the utility function (49) which satisfies $R_R(I_d) > 1$. Assuming for simplicity that $G = 1$ (thus regarding g as a proportion of time), we may use (53) to express the appropriate labor supply function as

$$g = \frac{\gamma}{\gamma + [\,(1-t)\,w\,]^{1/2}} \tag{53}$$

where $\gamma = \dfrac{p^{1/2} + (b-1)^{1/2}(1-p)^{1/2}}{b^{1/2}} < 1$ for $bp < 1$.

From (53)′ we now obtain t_A for the full-report economy, and t_B for the tax evasion economy

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[12] Note that for $bp < 1$ one has $\mu < 0$, so that $\delta > 1$.

$$t_A = 1 - \frac{(1-g_A)^2}{wg_A^{\ 2}} \tag{75}$$

$$t_B = 1 - \frac{\gamma^2 (1-g_B)^2}{wg_B^{\ 2}} \tag{76}$$

and substitute (75) and (76) into (74) to get, after rearranging terms [$\mu + \delta = 1$!], the following connection between g_A and g_B

$$w(g_B - g_A) = \delta\gamma^2 \frac{(1-g_B)^2}{g_B} - \frac{(1-g_A)^2}{g_A} \tag{77}$$

where $\delta\gamma^2 > 1$ (See Math. App. B).

Examining now the hypothesis that $g_B < g_A$, we should expect the right hand side of (77) to be non-positive since the left hand side is such. However, under the hypothesis above, the right hand side of (77) is necessarily positive since $\delta\gamma^2 > 1$. Hence we can not find such values for g_A and g_B which satisfy both (77) and $g_B < g_A$. Similarly, it can be seen that $g_B > g_A$ is a possible solution for (77). Hence, if there exist such values for g_A and g_B which equate the government's receipts from taxes and fines in economy A to its certain receipts from taxes in economy B, they necessarily satisfy the relation $g_B > g_A$. Assuming the utility function of (49), we thus conclude that under the constraint on governments' receipts, the total level of work in the economy where optimal tax evasion takes place exceeds that of the non-evasion economy.

For the utility function (47) which satisfies $R_R(I_d) < 1$, we may use now (51) to express the corresponding labor supply function as

$$g = \frac{(1-t)\,w}{(1-t)\,w + \beta} \tag{51}$$

where $\beta = \dfrac{b-1}{b[\,p^2\,(b-1) + (1-p)^2\,]} < 1$ for $bp < 1$.

From (51)' we obtain t_A and t_B as

$$t_A = 1 - \frac{g_A}{w\,(1-g_A)} \tag{78}$$

$$t_B = 1 - \frac{\beta g_B}{w(1 - g_B)} \tag{79}$$

and substitute (78) and (79) into (74) to get after rearranging terms, the following connection between g_A and g_B

$$w(g_B - g_A) = \beta\delta \left[\frac{g_B^2}{1 - g_B} - \frac{g_A^2}{1 - g_A} \right] \tag{80}$$

where $\beta\delta > 1$ (See Math. App. B).

It is apparent from (80) that $g_B \neq g_A$. However, it is obvious by examining (80) that we can not reject here the alternative hypotheses, so that contrary to the previous case there is no clear-cut relation between g_A and g_B when the utility function (47) is assumed. For example, supposing that $g_A = \frac{2}{5}$ we obtain for $w = 1$ and $\beta\delta = \frac{4}{3}$ the quadratic equation $35g_B^2 - 17g_B + 2 = 0$ the solutions for which are $g_B' = \frac{2}{7}$; $g_B'' = \frac{1}{5}$, so that $g_B < g_A$. On the other hand, for $w = 3$ and $\beta\delta = \frac{9}{8}$ we obtain the quadratic equation $495g_B^2 - 472g_B + 112 = 0$ which yields $g_B' = \frac{4}{9}$; $g_B'' = \frac{28}{55}$, so that $g_B > g_A$. One thus needs specific values for w and $\beta\delta$ to determine in this case whether g_B necessary to satisfy the initial constraint on governments's receipts exceeds or falls short of g_A.

VI. SUMMARY OF MAIN RESULTS

Along with allocating his time to work, the individual taxpayer is allowed in this model to expose himself to risk by reporting less than his actual income to the tax authorities. He is assumed to face given wage and tax rates, an exogenous probability of conviction, and a penalty function which is proportional to the taxes he evades. Formulating the necessary conditions for an optimum through maximizing his expected utility from leisure and disposable income, a comparative static analysis is carried out. It is shown that the way in which optimal solutions respond to changes in the parameters of the model depends upon the values of the Arrow–Pratt relative and absolute risk aversion measures. When relative risk aversion is a constant function of income, the results derived for the optimal proportion of actual income declared are similar to those obtainable by Yitzhak's model, since in this case the optimal proportion declared is independent of labor supply: A change in the market wage rate does not affect the proportion declared, but an increase in the income tax rate, penalty rate or the probability of conviction will tend to raise the proportion declared. When relative risk aversion varies with income, it is demonstrated that clear-cut results can not be derived without the application of specific utility functions. Using, for example, the exponential utility function for which absolute risk aversion is constant and relative risk aversion increases with income, it is found that an increase in the market wage rate, penalty rate, or the probability of conviction still raises the optimal proportion declared, as it does in a partial model where the allocation of time is given; But, contrary to the partial model where an increase in the income tax rate always raises the proportion of actual income declared, there exists now a domain of income tax rates for which the proportion declared will tend to fall.

Applying comparative static analysis to the optimal allocation of time, one should distinguish between three different cases when relative risk aversion is a constant function of income: If the relative risk aversion measure is less, equal, or greater than unity, an increase in the market wage rate will raise, leave constant or reduce time allocated to work, respectively, while an increase in the income tax rate, penalty rate or the probability of conviction will reduce, leave constant, or raise time allocated to work, respectively. Comparing this to the results obtained for the optimal proportion of income declared, it can be said that when relative risk aversion is constant, the optimal solutions respond in the same direction to changes in governmental parameters if the value of the measure exceeds unity, but in the opposite directions if its value is less than unity. As for a relative risk aversion which varies with income, it is shown again that unambiguous results can not be achieved unless specific utility functions are used. For the exponential utility function it is found that as long as the level of disposable income is sufficiently low so that the relative risk aversion measure is less than unity, an increase in the market wage rate will raise time allocated to work, while an increase in the income tax rate will reduce it. For a higher level of disposable income where the measure above exceeds unity, the reverse is expected to occur. An increase in the penalty rate or the probability of conviction will raise in this case the optimal allocation of time to work along with the optimal proportion declared.

Choosing specific utility functions for the three cases possible under the constant relative risk aversion assumption, the optimal proportion of actual income declared and allocation of time to work are explicitly derived. The "cash equivalent" of the uncertain disposable wage, which is a function of all four parameters, is shown to replace the legal disposable wage as an explanatory variable of labor supply. The explicit formulation of the "adjusted" labor supply function enables a comparison with the "original" full-report function, and verifies conclusions obtainable by a general diagramatic exposition. It is found that for given market wage and income tax rates, tax evasion is accompanied by an allocation of more, less or equal number of hours to work, if the relative risk aversion measure is less, greater or equal to unity, respectively. When relative risk aversion is an increasing function of income, an individual having an exponential utility function is found to allocate less hours to work when evading taxes. An increase in one of the government deterrent parameters – the penalty rate and the probability of conviction – will reduce in each case the gap between labor supply of tax evasion and that of full reporting of income.

Finally, a comparison is carried out, for the constant relative risk aversion case, between total level of work in a non-evasion economy to that taking place in a tax evasion economy. To compare between the two on an appropriate basis, it is assumed that the certain government's receipts from taxes in the first economy are equal to its expected receipts from taxes and fines in the other. It is found that if there exist such values for total work in both economies which satisfy this constraint, than for a relative risk aversion measure which exceeds unity, total level of work in an optimal tax evasion economy is necessarily higher than that taking place in a non-evasion economy. For a measure which equals unity, total levels of work are trivially equal, while for a relative risk aversion measure which is less than unity the total level of work in the first economy can exceed or fall short of that taking place in the second depending upon the specific values of the parameters involved in the model.

VII. MATHEMATICAL APPENDIX

$(A)_1$ *Deriving the Labor Supply Function for* $U = I_d^{1/2} + (G - g)^{1/2}$

From definition: $U(\phi g) = EU(I_d)$

Hence: $(\phi g)^{1/2} = (1 - p)[(1 - \epsilon t) wg]^{1/2} + p\left\{[1 - t(\epsilon + b(1 - \epsilon))] wg\right\}^{1/2}$

Using optimum condition (11): $\phi^{1/2} = (1 - p)[(1 - \epsilon t) w]^{1/2}\left[1 + \dfrac{1 - t(\epsilon + b(1 - \epsilon))}{(b - 1)(1 - \epsilon t)}\right]$

or: $\phi = \dfrac{w}{1 - \epsilon t} \cdot \left[\dfrac{b(1 - p)(1 - t)}{b - 1}\right]^2$

and substituting ϵ^* of (50): $\phi = (1 - t)w \dfrac{b[p^2(b - 1) + (1 - p)^2]}{(b - 1)}$

ϕ is thus independent of g. The maximization problem is:

$$\max_g \quad U(\phi g) + V(G - g)$$

and the first-order condition: $U'(\phi g^*)\phi = V'(G - g^*)$

so that by substituting the explicit function: $g^* = G \dfrac{\phi}{1 + \phi}$

$(A)_2$ Deriving the Labor Supply Function for $U = U_0 - \dfrac{1}{I_d} - \dfrac{1}{G - g}$

From definition: $U(\psi g) = EU(I_d)$

Hence: $\dfrac{1}{\psi g} = \dfrac{1 - p}{(1 - \epsilon t) wg} + \dfrac{p}{[1 - t(\epsilon + b(1 - \epsilon))] wg}$

Using optimum condition (11): $\dfrac{1}{\psi} = \dfrac{1 - p}{(1 - \epsilon t) w}\left[1 + \dfrac{1 - t(\epsilon + b(1 - \epsilon))}{(b - 1)(1 - \epsilon t)}\right]$

or: $\psi = \dfrac{(b-1)\, w\, (1-\epsilon t)^2}{b\,(1-p)(1-t)}$

and substituting ϵ^* of (50): $\psi = (1-t)\, w \dfrac{b}{[\, p^{1/2} + (b-1)^{1/2}(1-p)^{1/2} \,]^2}$

ψ is thus independent of g. The maximization problem is:

$$\max_{g} \quad U(\psi g) + V(G-g)$$

and the first-order condition: $U'(\psi g^*)\, \psi = V'(G-g^*)$

so that by substituting the explicit function: $g^* = G \dfrac{1}{1+\psi^{1/2}}$

$(A)_3$ Deriving the Labor Supply Function for $U = U_0 - e^{-I_d} - e^{-(G-g)}$

From definition: $U(\lambda g) = EU(I_d)$

Hence: $e^{-\lambda g} = (1-p)\, e^{-(1-\epsilon t) wg} + p e^{-[1 - t(\epsilon + b(1-\epsilon))] wg}$

Using optimum condition (11): $e^{-\lambda g} = (1-p)\, e^{-(1-\epsilon t) wg} \left(1 + \dfrac{1}{b-1}\right)$

or: $\lambda = (1-\epsilon t)\, w - \dfrac{1}{g} \ln \dfrac{(1-p)\, b}{b-1}$

and substituting ϵ^* of (55): $\lambda = (1-t)\, w + \dfrac{1}{g} \left[\dfrac{1}{b} \ln \dfrac{1-p}{p\,(b-1)} - \ln \dfrac{(1-p)\, b}{b-1} \right]$

λ is thus a function of g. The maximization problem is:

$$\max_{g} \quad U[\lambda(g)\, g] + V(G-g)$$

and the first-order condition: $U'[\lambda(g^*)\, g^*][\lambda(g^*) + g^* \lambda'(g^*)] = V'(G-g^*)$

so that by substituting the explicit function: $g^* = \dfrac{G + \ln(1-t)\, w}{1 + \lambda(g^*)}$

$(B)_1$ Proof of $\delta \gamma^2 > 1$ for $U = U_0 - \frac{1}{I_d} - \frac{1}{G-g}$

From definition: $\delta = \frac{b(1-p)^{1/2}(1-bp)}{(b-1)^{1/2}[p^{1/2} + (b-1)^{1/2}(1-p)^{1/2}]} + bp > 1$

$$\gamma = \frac{p^{1/2} + (b-1)^{1/2}(1-p)^{1/2}}{b^{1/2}} < 1$$

Hence: $\delta\gamma^2 = \frac{[(1-p)^{3/2} + p^{3/2}(b-1)^{1/2}][p^{1/2} + (b-1)^{1/2}(1-p)^{1/2}}{(b-1)^{1/2}}$

We have to show that:

$$(1-p)^{3/2} p^{1/2} + (b-1)^{1/2}(1-p)^{1/2} + p^2 (b-1)^{1/2} + p^{3/2}(b-1)(1-p)^{1/2} > (b-1)^{1/2}$$

expanding $(1-p)^2$ and rearranging:

$$(1-p)^{3/2} p^{1/2} + p^{3/2}(b-1)(1-p)^{1/2} > 2p(1-p)(b-1)^{1/2}$$

Dividing by $p^{1/2}(1-p)^{1/2}$:

$$(1-p) + p(b-1) > 2p^{1/2}(1-p)^{1/2}(b-1)^{1/2}$$

so that by squaring both sides we get:

$$1 - 2bp + b^2p^2 = (1-bp)^2 > 0.$$

$(B)_2$ Proof of $\beta\delta > 1$ for $U = I_d^{1/2} + (G-g)^{1/2}$

From definition: $\delta = \frac{b(1-p)^2(1-bp)}{(b-1)[p^2(b-1) + (1-p)^2]} + bp > 1$

$$\beta = \frac{b-1}{b[p^2(b-1) + (1-p)^2]} < 1$$

Hence: $$\beta\delta = \frac{p^3 (b-1)^2 + (1-p)^3}{[p^2 (b-1) + (1-p)^2]^2}$$

We have to show that:

$$p^3 (b-1)^2 + (1-p)^3 > p^4 (b-1)^2 + 2p^2 (b-1)(1-p)^2 + (1-p)^4$$

Rearranging: $p^3 (b-1)^2 (1-p) + p(1-p)^3 > 2p^2 (b-1)(1-p)^2$

Divided by $p(1-p)$: $p^2 (b-1)^2 + (1-p)^2 > 2p(b-1)(1-p)$

So that by expanding and eliminating terms we get:

$$1 - 2bp + b^2 p^2 = (1-bp)^2 > 0.$$

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