THE NATIONAL INSURANCE INSTITUTE
BUREAU OF RESEARCH AND PLANNING

EQUIVALENCE SCALES FOR FAMILY SIZE:
FINDINGS FROM ISRAELI DATA

By
Jack Habib and Yossi Tawil

Draft
Not for Quotation
Comments Invited

DISCUSSION PAPER 1

JERUSALEM, MARCH 1974

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ACKNOWLEDGEMENT

This is the first in a series of discussion papers designed to subject research being conducted at the Institute to criticism and comment and to promote a dialogue with others involved in similar areas of research and policy-making.

Dr. Habib is presently director of the Division of Long-Range Research and Yossi Tawil is a senior economist in this division.

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There has been growing recognition that in measuring the extent of poverty and inequality, determining the redistributive impact of taxes and transfers or evaluating redistributional schemes, one must allow for demographic differences among families such as age composition and family size. In order to allow for these factors it is necessary to establish a basis for the comparison of the living standards of families of different demographic types. In other words, one must be able to define incomes at which the living standard of a family of type A will be equivalent to that of a family of type B. One approach is to define an equivalence scale in terms of standard persons with a given family type as a base.

An equivalence scale for differences in family size was estimated at the National Insurance Institute in Israel in 1970, and has since been widely used both in research and in policy making.[1] This scale, which is presented in Table 1 (hence official scale) has two essential features. It presumes that there are economies of scale in family consumption i.e. that a child requires less than an adult and that the needs of each additional child decline; and economies of scale are presumed to be the same at all levels of family income, i.e. the relative needs of additional family members do not vary with income.[2]

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1 See for example, Benjamin Bridges, Jr., "Family Need Differences and Family Tax Burden Estimates," *National Tax Journal*, XXIV (December 1971) 423-43; Joseph J. Seneca and Michael K. Taussig, "Family Equivalence Scales and Personal Income Tax Exemptions for Children," *Review of Economics and Statistics*, LIII (August 1971) 253-62; and E. Kleiman, "Age Composition, Size of Household, and the Interpretation of Per Capita Income," *Economic Development and Cultural Change*, XV (October 1966) 37-58; and *idem*, "A Standardized Dependency Ratio," *Demography*, IV (No. 2 1967) 876-93; for Israel see R. Roter and N. Shamai, "Patterns of Poverty in Israel - Preliminary Findings," *Social Security* (Hebrew with English summary), No. 1 (February 1971), pp. 17-28; *Report of the Committee on Income Distribution and Social Inequality* (Tel Aviv: 1971); Joseph Gabbay, "Comparison of the Incidence of the Tax System by Families and by Standard Adults" (paper presented at the conference on Issues in the Economics of Israel, Jerusalem, April 1973; Hebrew); Jack Habib, *The Role of Child Allowances in a Tax-Transfer Structure* (Jerusalem: Falk Institute, 1972); *idem*, *Children in Israel* (Research Report No. 168; Jerusalem: Szold Institute, 1972).

2 Any family size may be arbitrarily chosen as the base for this scale. Throughout this article the base is a family of two persons.

TABLE 1: The Official Scale

	Family Size (Number of Persons)								
	1	2	3	4	5	6	7	8	9
No. of Standard Persons	1.25	2.00	2.65	3.20	3.75	4.25	4.75	5.20	5.60
Weight of Additional Person	-	0.75	0.65	0.55	0.55	0.50	0.50	0.45	0.40

See *Report of the Committee on Income Distribution and Social Inequality* (Tel Aviv: 1971), Appendix A. Table, p. 39.

The most common technique for estimating these scales was first suggested by Friedman and was subsequently elaborated by David and Watts. The technique is based on the assumption that families which consume a given percentage of their income on a defined market basket are equally well off (at least from the point of view of real consumption) irrespective of family size or composition.[3] We have shown elsewhere that this assumption is essentially arbitrary and will be correct for only a very limited concept of family equivalence and for equally limiting assumptions with respect to the family welfare function.[4] Moreover, even if this assumption is accepted, it is still necessary to determine arbitrarily

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[3] See Milton Friedman, "A Method of Comparing Incomes of Families Differing in Composition," *Studies in Income and Wealth*, Vol. 15 (New York: National Bureau of Economic Research, 1952), pp. 9-20; Martin David, "Welfare, Income, and Budget Needs," *Review of Economics and Statistics*, XLI (November 1959), 393-99; Harold W. Watts, "The ISO-Prop Index: An Approach to the Determination of Differential Poverty Income Thresholds," *Journal of Human Resources*, II (No. 1, Winter 1967), 3-18. For some recent application of this approach see Seneca and Taussig, *op.cit.* and Elliot Wetzler, *Determination of Poverty Lines and Equivalent Welfare* (Institute for Defense Analysis, Economic and Political Studies Division, Research Paper P-277; Washington D.C., 1966). For summary of the literature see Carolyn A. Jackson, *Revised Equivalence Scales for Estimating Equivalent Incomes or Budget Costs by Family Type* (U.S. Department of Labor, Bureau of Labor Statistics: Bulletin No. 1570-2; Washington, D.C. 1968).

[4] See Jack Habib, *"The Determination of Equivalence Scales with Respect to Family Size: A Theoretical Reappraisal,"* (Jerusalem: Falk Institute, 1973).

what items are to be included in the market basket and to choose among a number of procedures for estimating the income levels at which the share of expenditure on these items is equal among family types.

In this paper we are interested in testing the sensitivity to the definition of the market basket and to the estimating procedure, of estimates of equivalence scales for family size. We examine the possible range of economies or diseconomies of scale and whether they vary with family income. If they do so vary, we are particularly interested in determining whether the scale currently in use in Israel over or under-estimates the extent of economies in low income ranges. This will provide us with an indication of the direction of the bias in existing estimates of the extent of poverty and of the composition of the poor which have made use of this scale.

FORM OF THE CONSUMPTION FUNCTION

The estimation of equivalence scales is based on the prior estimation of a functional relationship between expenditures on a given basket, income and family characteristics, commonly referred to as the consumption function. Therefore one of the most important decisions to be made, is with respect to the form of the function. In empirical studies of consumption, various functional forms have been proposed. We experimented with a number of possibilities and rejected those that yielded unreasonable results, The criterion employed was that the scales derived from our estimates satisfy the condition that the well being of a family at a given level of money income declines with an increase in family size and that at the other extreme the number of standard persons not exceed the number of persons in the family to an unreasonable extent.

We present results for two functions, the Cobb-Douglas (log linear function - hence C-D) $E=AY^{b_1}N^{b_2}$, and a quadratic function $E=A+b_1Y+b_2Y^2+b_3N$, where E is expenditure on a given market basket, Y is the measure of family resources and N is the number of persons in the family.

The major difference between the two functions is that in the Cobb-Douglas function the expenditure elasticities with respect to family size and income are constant while with the quadratic form these elasticities are permitted to vary with income and family size. As a consequence economies of scale will not vary with income for the Cobb-Douglas function, while with the quadratic such variation is possible. The direction and extent of the variation is determined by the value of the estimated elasticities. The derivation of the scale for each function is presented in Appendix A.

The quadratic function, however, has certain statistical disadvantages as a result of the high correlation between Y and Y^2. It may therefore be useful to employ other methods for testing the relationship between income and economies of scale. One possibility, which was used by Seneca

and Taussig, is to estimate separate consumption functions for each family size.[5] We tried to employ this method, however, because of the limited size of our sample we did not obtain significant results.

THE CHOICE OF A MARKET BASKET

There is an a-priori basis for postulating that the opportunities for economies or savings as family size increases, differ between goods. This has been confirmed by previous studies of consumption patterns. For this reason the estimates of the equivalence scale will be influenced by the choice of items included in the market basket. In order to examine the possible range of economies of scale we make use of three alternative definitions: Expenditure on food (F); on food and clothing (F+C); on food, clothing and housing (F+C+H).

THE MEASURE OF FAMILY RESOURCES

Previous studies have used pre-tax income as the basis for estimating equivalence scales. However, the use of pre-tax income provides a distorted view of the relationship between income, expenditure and family size since the structure of taxation significantly alters the relative incomes of families by initial incomes and family size. For example, the average tax rate at a given income level declines with family size. As a result the estimated response of expenditure to changes in family size when gross income is held constant is biased, since it includes the effect of the rise in disposable income accompanying the increase in family size. We have therefore chosen to base our estimates on a measure of after tax resources. In this measure we have also included the imputed value of services from the owner-ship of a home or motor vehicle.

Another problem is how to differentiate between transitory and permanent components in the measure of both resources and the family's expenditure on a particular market basket.

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[5] See Seneca and Taussig *Op. Cit.* Their results support the hyphothesis of variation in the scale with income.

One solution that has been proposed is the use of the household's total consumption expenditure as a proxy for permanent income. In support of this measure it is argued that the transitory component is smaller in consumption expenditure than in disposable income (as households attempt to smooth out their consumption expenditure by allowing savings to obsorb much of the temporary fluctuation in income). Liviatan however shows that under certain conditions the use of total consumption expenditure as a proxy for permanent income involves a bias in the estimate of the true coefficient.[6]

Two alternative solutions that have been suggested are data grouping and the use of an instrumental variable. Liviatan shows that by grouping the data according to disposable income, and using for the independent variable, the average total consumption expenditure in each group, a consistent estimate is obtained. We have adopted this procedure, basing our estimates on a sample of 2200 urban Jewish families drawn from the 1968/69 Family Expenditure survey.[7] These families have been devided into ten disposable income groups and nine family size groups, yielding ninety income-family size cells. Due to insufficient observations in the upper income -large family cells only eighty eight of the possible combinations are employed. The regression results are presented in Tables 2 and 3.

The results indicate that the functions fit the data well, as the adjusted correlation coefficient exceeds 0.90. Moreover, most of the estimated coefficients are significant at the 1 percent level. The coefficient of income squared (b_2) in the quadratic function is significant at a close to 5 percent level. The significance of this coefficient decreases with the expansion of the market basket as a-priori, reasoning would suggest. The reason for this is that with the addition of consumption items, the difference between total consumption and the expenditure

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[6] See Nissan Leviatan, Consumption Patterns in Israel. (Jerusalem: Falk Project 1964); *idem*, "Errors in Variables and Engel Curve Analysis," *Econometrica*, July 1961; Robert T. Michael, *The Effect of Education on Efficiency in Consumption* (National Bureau of Economic Research: Occasional Paper No. 116; New York, 1972), Chapter 2.

[7] Central Bureau of Statistics, *Family Expenditure Survey 1968/69 Part A* (Special Series No. 330; Jerusalem: 1970).

TABLE 2: Regression Coefficients Based on Cobb-Douglas Consumption Function

Dependent Variable Log E[a]	Constant Log A	Income[b] Log Y	Number of persons Log N	Adjusted R^2
Log (F)	1.72 (0.17)[c]	0.49 (0.03)	0.40 (0.01)	0.95
Log (F+C)	1.08 (0.19)	0.64 (0.03)	0.34 (0.02)	0.94
Log (F+C+H)	1.53 (0.13)	0.67 (0.02)	0.15 (0.01)	0.95

TABLE 3: Regression Coefficients Based on Quadratic Consumption Function

Dependent Variable / E[a]	Constant A	Income[b] Y	Income[b] Squared Y^2	Number of persons N	Adjusted R^2
F	-6.08 (22.58)[c]	0.20 (0.05)	-0.000038 (0.000018)	27.55 (1.33)	0.91
F+C	-30.99 (32.28)	0.32 (0.07)	-0.000036 (0.000020)	31.58 (1.91)	0.90
F+C+H	52.69 (29.74)	0.45 (0.06)	-0.000028 (0.000016)	23.68 (1.76)	0.94

[a] F = expenditure on food
C = expenditure on clothing
H = expenditure on housing

[b] See footnote 8

[c] Standard deviation

on the market basket diminishes, and the link between the two, is increasingly linear.[8]

FINDINGS

In this section, we present the scales with respect to family size, as derived on the basis of the estimated consumption functions. We begin by summarizing our major findings:

1) For both functions the estimate of economies of scale is highly sensitive to the definition of the market basket. The pattern is consistent: economies are greatest for food and clothing and smallest for food, clothing and housing.

2) With the C-D there exist economies of scale for all market baskets.

3) For the quadratic, economies of scale rise significantly and at an increasing rate with an increase in income. On the basis of food or food and clothing there are diseconomies of scale over a wide range of incomes.

4) The C-D yields an overestimate of scale economies at low incomes and an underestimate at higher incomes in comparison with the quadratic.

5) The official scale overestimates scale economies for low income families relative to scales based on food or food and clothing and underestimates scale economies relative to scales based on food clothing and housing.

RESULTS FOR FOOD EXPENDITURE

We compare the scales obtained from the C-D and quadratic function for food expenditures. The scales for C-D are presented in Table 4 and for the quadratic in Table 5.

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[8] We henceforth refer to total consumption as "income" since we employ it as an indicator of "true" income. This is customary in studies of consumer expenditure.

TABLE 4: Equivalence Scales on the Basis of the Cobb-Douglas Function and Alternative Market Baskets

No. of persons	Food		Food and Clothing		Food, Clothing and Housing	
	No. of Standard Persons	Weight of Additional Persons	No. of Standard Persons	Weight of Additional Persons	No. of Standard Persons	Weight of Additional Person
1	1.16	-	1.04	-	1.46	-
2	2.00	0.84	2.00	0.96	2.00	0.54
3	2.75	0.75	2.93	0.93	2.40	0.40
4	3.44	0.69	3.85	0.92	2.74	0.34
5	4.10	0.66	4.75	0.90	3.03	0.29
6	4.73	0.63	5.64	0.89	3.29	0.26
7	5.34	0.61	6.53	0.89	3.53	0.24
8	5.93	0.59	7.40	0.87	3.75	0.22
9	6.50	0.57	8.27	0.87	3.96	0.21

TABLE 5: Equivalence Scale Derived on the Basis of the Quadratic Function for Food Expenditures

No. of Persons	Y=100[a/]	Y=200	Y=300	Y=400	Y=500	Y=600	Y=700	Y=800	Y=1000	Y=1200	Y=1400	Y=1600	Y=2000
No. of Standard Persons													
1	0.88	0.89	0.90	0:92	0.95	0.99	1.03	1.07	1.17	1.28	1.39	1.48	1.62
2	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
3	3.11	3.08	3.04	2.98	2.92	2.86	2.79	2.73	2.62	2.53	2.45	2.39	2.30
4	4.22	4.14	4.02	3.89	3.75	3.61	3.48	3.36	3.15	2.97	2.83	2.72	2.55
5	5.32	5.17	4.97	4.74	4.51	4.29	4.09	3.91	3.60	3.36	3.16	3.01	2.77
6	6.41	6.18	5.87	5.54	5.21	4.92	4.65	4.41	4.02	3.71	3.46	3.26	2.97
7	7.49	7.17	6.74	6.30	5.88	5.50	5.17	4.88	4.40	4.03	3.73	3.50	3.16
8	8.57	8.13	7.58	7.02	6.50	6.05	5.65	5.31	4.75	4.32	3.99	2.72	3.33
9	9.64	9.08	8.39	7.71	7.10	6.57	6.11	5.71	5.08	4.60	4.23	3.93	3.49
Weight of Additional Person													
2	1.12	1.11	1.10	1.08	1.05	1.01	0.97	0.93	0.83	0.72	0.61	0.52	0.38
3	1.11	1.08	1.04	0.98	0.92	0.86	0.79	0.73	0.62	0.53	0.45	0.39	0.30
4	1.11	1.06	0.98	0.91	0:83	0.75	0.61	0.63	0.53	0.44	0.38	0.33	0.25
5	1.10	1.03	0.95	0.85	0.76	0.68	0.61	0.55	0.45	0.39	0.33	0.29	0.22
6	1.09	1.01	0.90	0.80	0.70	0.63	0.56	0.50	0.42	0.35	0.30	0.25	0.20
7	1.08	0.99	0.87	0.76	0.67	0.58	0.52	0.47	0.38	0.32	0.27	0.24	0.19
8	1.08	0.96	0.84	0.72	0.62	0.55	0.48	0.43	0.35	0.29	0.26	0.22	0.17
9	1.07	0.95	0.81	0.69	0.60	0.52	0.46	0.40	0.33	0.28	0.24	0.21	0.16

[a/] Income for a family of size two (Y).

The C-D function suggests the existence of considerable scale economies that increase with each additional family member. Thus the needs of the first child are 75% of those of an adult and the needs of subsequent children decline to 60% for the seventh or eighth child.

With the quadratic function there is marked variation in economies of scale with the level of income. At low incomes there are diseconomies. Thus the requirements of the first child exceed those of an adult at income levels up to I.L.400 and for all subsequent children up to incomes in the range of I.L.300. As income rises the extent of diseconomies declines (economies rise). For example, at an income level of 800 the additional needs of the first child decline to 73% of those of an adult and at an income level of 2000 to only 30%. In interpreting these results one must keep in mind that a family of two persons serves as the base for the scale. As a result the indicated income levels are also for a family of two persons.[9]

One way of comparing the results for the C-D and quadratic, is in terms of the income level at which the degree of economies is similar. We find that this income level tends to decline with family size and in all cases is in the vicinity of 600-700 as seen in Table 6. This income level is somewhat below average monthly disposable family income of 754 in 1969.

TABLE 6: Comparison of Equivalence Scales for Cobb-Douglas and Quadratic Functions for Food Expenditures.

Number of Persons	Cobb-Douglas	Quadratic		
		Y=600[a/]	Y=700	Y=800
4	3.44	3.61	3.48	3.36
8	5.93	6.05	5.65	5.31

Source: Tables 4 and 5

a/ Income for a family of size two (Y)

[9] In addition the incomes are expressed in 1969 prices. Economies of scales are influenced by a number of factors that are likely to change over time, such as relative prices and consumption patterns. There is therefore no certain way of expressing the scales in terms of current income levels.

This is as expected since the majority of families have incomes below the average. Below this level the C-D provides an overestimate of scale economies and above this level it provides an underestimate. Furthermore, these differences are quite considerable at many income levels.

ALTERNATIVE MARKET BASKETS

The effects of altering the definition of the market basket have a consistent pattern. For either the C-D or the quadratic, economies are smallest with food and clothing and are greatest with food, clothing and housing. Examining the range of variation for the C-D, we find that all three baskets yield economies of scale but that the differences are considerable. For the C-D the results are compared in Table 4. We see that the number of standard adults for a family of 4 is 3.44 on the basis of food expenditure, 3.85 on the basis of food and clothing, and 2.74 on the basis of food, clothing and housing. For a family of 8, the number of standard adults is 5.93 on the basis of food expenditure, it is 7.40 when based on food and clothing and 3.75 when based on food, clothing, and housing expenditures.

The results for the quadratic are found in Tables 5, 7 and 8 and the three baskets are compared in Table 9.

TABLE 9: The Degree of Economies of Scale with Respect to Family Size Based on the Quadratic Function and on Three Alternative Baskets

	4 persons			8 persons		
	Y=300[a]	Y=800	Y=1400	Y=300	Y=800	Y=1400
Basket						
F	4.02	3.36	2.83	7.58	5.31	3.99
F+C	5.22	3.83	3.04	10.24	6.15	4.36
F+C+H	2.92	2.80	2.63	4.72	4.23	3.66

[a] Income for a family of size two.

Source: Tables 5,7 and 8

TABLE 7: Equivalence Scales Derived on the Basis of the Quadratic Function for Food and Clothing Expenditures

No. of Persons	Y=100[a/]	Y=200	Y=300	Y=400	Y=500	Y=600	Y=700	Y=800	Y=1000	Y=1200	Y=1400	Y=1600	Y=2000
No. of Standard Persons													
1	0.04	0.04	0.04	0.04	0.05	0.05	0.07	0.09	0.22	0.60	0.95	1.19	1.48
2	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
3	3.93	3.83	3.70	3.55	3.41	3.27	3.03	3.03	2.84	2.70	2.58	2.49	2.36
4	5.82	5.56	5.22	4.88	4.56	4.28	3.83	3.83	3.49	3.24	3.04	2.88	2.66
5	7.69	7.19	6.61	6.06	5.57	5.16	4.51	4.51	4.04	3.69	3.43	3.22	2.92
6	9.52	8.75	7.90	7.13	6.48	5.93	5.11	5.11	4.53	4.09	3.77	3.51	3.14
7	11.32	10.24	9.10	8.11	7.30	6.64	5.65	5.65	4.96	4.46	4.08	3.78	3.35
8	13.10	11.67	10.24	9.04	8.07	7.30	6.15	6.15	5.36	4.79	4.36	4.03	3.55
9	14.85	13.05	11.32	9.90	8.79	7.91	6.62	6.62	5.74	5.10	4.63	4.26	3.73
Weight of Additional Person													
2	1.96	1.96	1.96	1.96	1.95	1.95	1.93	1.91	1.78	1.40	1.05	0.81	0.52
3	1.93	1.83	1.70	1.55	1.41	1.27	1.14	1.03	0.84	0.70	0.58	0.49	0.36
4	1.89	1.73	1.52	1.33	1.15	1.01	0.90	0.80	0.65	0.54	0.46	0.39	0.30
5	1.87	1.63	1.39	1.18	1.01	0.88	0.77	0.68	0.55	0.45	0.39	0.34	0.26
6	1.83	1.56	1.29	1.07	0.91	0.77	0.68	0.60	0.49	0.40	0.34	0.29	0.22
7	1.80	1.49	1.20	0.98	0.82	0.71	0.61	0.54	0.43	0.37	0.31	0.27	0.21
8	1.78	1.43	1.14	0.93	0.77	0.66	0.57	0.50	0.40	0.33	0.28	0.25	0.20
9	1.75	1.38	1.08	0.86	0.72	0.61	0.53	0.47	0.38	0.31	0.27	0.23	0.18

[a/] Income for a family of size two (Y)

TABLE 8: Equivalence Scales Derived on the Basis of the Quadratic Function for Food, Clothing and Housing Expenditures

No. of Persons	Y=100[a/]	Y=200	Y=300	Y=400	Y=500	Y=600	Y=700	Y=800	Y=1000	Y=1200	Y=1400	Y=1600	Y=2000
No. of Standard Persons													
1	1.53	1.53	1.53	1.54	1.54	1.55	1.56	1.57	1.58	1.62	1.64	1.67	1.72
2	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
3	2.47	2.47	2.46	2.46	2.45	2.44	2.42	2.41	2.40	2.35	2.33	2.30	2.48
4	2.94	2.94	2.92	2.90	2.88	2.86	2.93	2.80	2.77	2.68	2.63	2.57	2.48
5	3.42	3.40	3.38	3.35	3.31	3.27	3.23	3.18	3.13	2.99	2.91	2.82	2.68
6	3.89	3.86	3.83	3.79	3.73	3.67	3.61	3.54	3.48	3.29	3.17	3.06	2.88
7	4.36	4.33	4.28	4.22	4.14	4.06	3.98	3.89	3.81	3.56	3.42	3.29	3.06
8	4.83	4.79	4.72	4.64	4.55	4.45	4.34	4.23	4.13	3.83	3.66	3.50	3.24
9	5.30	5.25	5.17	5.07	4.95	4.82	4.69	4.56	4.44	4.09	3.88	3.70	3.40
Weight Additional Person													
2	0.47	0.47	0.47	0.46	0.46	0.45	0.44	0.43	0.42	0.38	0.36	0.33	0.28
3	0.47	0.47	0.46	0.46	0.45	0.44	0.42	0.41	0.40	0.35	0.33	0.30	0.25
4	0.47	0.47	0.46	0.44	0.44	0.42	0.41	0.39	0.37	0.33	0.30	0.27	0.23
5	0.48	0.46	0.46	0.45	0.43	0.41	0.40	0.38	0.36	0.31	0.28	0.25	0.20
6	0.47	0.46	0.45	0.44	0.42	0.40	0.39	0.36	0.35	0.30	0.25	0.24	0.20
7	0.47	0.47	0.45	0.43	0.41	0.39	0.37	0.35	0.33	0.27	0.25	0.23	0.18
8	0.47	0.46	0.44	0.42	0.41	0.39	0.36	0.34	0.32	0.27	0.24	0.21	0.18
9	0.47	0.46	0.45	0.43	0.40	0.37	0.35	0.33	0.31	0.26	0.22	0.20	0.16

[a/] Income for a family of size two (Y).

For all definitions of the basket economies of scale rise with income and this variation is considerable. At the same time there are also highly significant differences in the scales obtained from each basket. Where as for F+C+H there are economies of scale at all income levels, for F or F+C, there is a considerable range of low incomes at which there are diseconomies. These diseconomies are strongest with F+C and prevail over an extensive range of incomes. Thus at the bottom of the income scale the needs of the first child are almost twice those of an adult and they continue to exceed those of an adult up to an income of I.L.800.

The comparison between the C-D and the quadratic is also similar for each basket in that at lower incomes the C-D overestimates economies and at higher incomes the C-D underestimates economies. The point of switchover is similar for food or food and clothing while with food, clothing and housing it occurs at a considerably higher income.

It should be pointed out that the notion of economies of scale has more than one dimension and that it is important to distinguish the relative needs of children and adults from the relative needs of additional children. Thus we find that the needs of each additional child uniformly decline on the basis of all baskets and functions, despite the fact that they sometimes exceed those of an adult. The implication is that while there are income levels at which there are <u>diseconomies</u> for families with children relative to childless couples, there may at the same income be <u>economies</u> of scale among families with children.

One way of illustrating this point is by expressing the existence of economies or diseconomies in terms of the pattern of per-capita income. The existence of economies or diseconomies between any two family sizes will be reflected by whether the larger family requires a lower, higher or equal level of per capita income to have an equivalent living standard. We may define complete scale economies as the situation in which the required per capita income falls continuously as family size rises.

This situation will be obtained when the needs of an additional child are less than the average needs per family member.

In order to summarize our findings we compare in Table 10 the equivalent incomes by family size that are consistent with the poverty line employed in 1969 by the National Insurance Institute set at a level of I.L.200 for a family of 2 adults.[10]

Economies of scale are less than those of the official scale for either function on the basis of food or food and clothing while they are greater for food, clothing and housing. The implication is that for the first two market baskets the estimated number of poor families with children would be larger than the estimates yielded by the official scale. If these scales were used as a criteria for income support it is clear that the transfer to families with children would be considerably larger. In the case of each market basket scale economies are lower for the quadratic than with the C-D. For F and F+C, the poverty line per-capita rises up to a family size of 5-6 persons and is then nearly constant.[11] The result that is closest to the official scale is obtained for the C-D estimate of food expenditures. This is as expected since the official scale was estimated on a similar basis from non-grouped observations.

QUALIFICATIONS

In estimating the consumption function there are a number of additional demographic variables that we have not taken into consideration such as age structure or ethnic affiliation. In addition we have not considered the role of the pattern of family labour force participation, e.g. spouse's employment. It is not always desireable to take these factors into account in designing policy. Leaving them out of the regression, however, may bias the estimates of the income and family size elasticities. For example ethnic affiliation appears to have a significant independent effect on expenditure pattern in Israel and is strongly correllated with family size and income. However given the weakness of the basic assumption which underlies the determination of equivalence, there is some doubt as to the profitability of further refinements along the same lines. We may

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[10] IL 200 for a family of two is equivalent to IL 100 p.s.p. The equivalent incomes are thus obtained by multiplying the number of standard persons for each family size by 100.

[11] As previously explained the marginal needs of each additional child decline leading eventually to the near constancy of per-capita needs.

TABLE 10: Poverty Line by Family Size Based on the Minimum Income of I.L.200 per couple and Alternative Estimates of the Equivalence Scale

Number of Persons	Cobb-Douglas			Quadratic			Offcial Scale
	F	F+C	F+C+H	F	F+C	F+C+H	
Poverty line income per family:							
1	116	104	146	89	4	153	125
2	200	200	200	200	200	200	200
3	275	293	240	308	383	247	265
4	344	385	274	414	556	294	320
5	410	475	303	517	719	340	375
6	473	564	329	618	875	386	425
7	534	653	353	717	1024	433	475
8	593	740	375	813	1167	479	520
Poverty line income per capita:							
1	116	104	146	89	4	153	125
2	100	100	100	100	100	100	100
3	92	98	80	103	128	82	88
4	86	96	69	104	139	74	80
5	82	95	61	103	144	68	75
6	79	94	55	103	146	64	71
7	76	93	50	102	146	62	68
8	74	93	47	102	146	60	65

have to reconcile ourselves with our inability to compare demographic groups on any certain bases. The implication is that in studies of trends in inequality and poverty we may have to distinguish between changes within and between demographic groups, and test the sensitivity of our results to a range of assumptions about their relative needs.

APPENDIX: DERIVATION OF EQUIVALENCE SCALES

In this appendix we describe the method employed to derive the equivalence scale from an estimated consumption function.

COBB-DOUGLAS CONSUMPTION FUNCTION

This function is of the form

$$E = AY^{b_1}N^{b_2}$$

for this function b_1 is the elasticity of expenditure on commodity E with respect to income. b_2 is the elasticity with respect to family size. These elasticities are constant.

Consider two families i and j of different size (N) where $N_j > N_i$. The basic assumption is that families which spend an equal percentage of their income on a given market basket have equivalent living standards. Thus i and j are equal when

$$(1)\quad \frac{E_i}{Y_i} = AY_i^{(b_1-1)}N_i^{b_2} = AY_j^{(b_1-1)}N_j^{b_2} = \frac{E_j}{Y_j}$$

from (1) we obtain that the required ratio of income is

$$(2)\quad \frac{Y_j}{Y_i} = \left(\frac{N_i}{N_j}\right)^{\frac{b_2}{b_1-1}}$$

Defining K as the index of equivalent need units then when (1) is satisfied:

$$(3)\quad \frac{K_j}{K_i} = \frac{Y_j}{Y_i}$$

Let the base family size equal 2 persons, then setting $N_i=2$, implies $K_i=2$. Substituting from (2) into (3) we obtain an expression for the number of standard persons for family of any other size

$$(4)\quad K_j = \frac{2Y_j}{Y_i} = 2\left(\frac{2}{N_j}\right)^{\frac{b_2}{b_1-1}}$$

from (4) it follows that the equivalence scale is a function only of the income and family size elasticities and is independent of family income.

We directly derive the condition for economies of scale with respect to the base family size.

for $N_j > 2$

$$2 < K_j < N_j \text{ if } -1 < \frac{b_2}{b_1 - 1} < 0$$

and

$$K_j > N_j \quad \text{if} \quad \frac{b_2}{b_1 - 1} < -1$$

In more general terms

$$\frac{\partial (K/N)}{\partial N} \gtrless 0 \text{ if } \frac{b_2}{b_1 - 1} \lessgtr -1$$

QUADRATIC CONSUMPTION FUNCTION

This function is of the form

$$E = A + b_1 Y + b_2 Y^2 + b_3 N$$

where $\eta_{EY} = \dfrac{(b_1 + 2b_2 Y)Y}{A + b_1 Y + b_2 Y^2 + b_3 N}$ and

$$\eta_{EN} = \frac{b_3 N}{A + b_1 Y + b_2 Y^2 + b_3 N}$$

As can be seen, both elasticities are a function of income and family size.

Letting $\dfrac{E_i}{Y_i} = \dfrac{E_j}{Y_j}$ we obtain

$$(1) \quad \frac{A + b_2 Y_i^2 + b_3 N_i}{Y_i} = \frac{A + b_2 Y_j^2 + b_3 N_j}{Y_j}$$

Assuming $Y_i ; \ Y_j \neq 0$, (1) may be written

as: (2) $AY_j + b_2 Y_i^2 Y_j + b_3 N_i Y_j = AY_i + b_2 Y_j^2 Y_i + b_3 N_j Y_i$

and by transfering terms we obtain

(3) $b_2 Y_i Y_j^2 - (A + b_2 Y_i^2 + b_3 N_i) Y_j + (A + b_3 N_j) Y_i = 0$

Solving this quadratic we find that Y_j is a function of Y_i, N_i and N_j and using $K_j = 2 \frac{Y_j}{Y_i}$ we see that the equivalence scale is dependent on the level of family income, i.e. $K_j = f(Y_i, N_i, N_j)$.

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