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THE NATIONAL INSURANCE INSTITUTE

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THE OPTIMAL LINEAR INCOME-TAX : A NOTE ON UNEMPLOYMENT COMPENSATION

by

Gideon Yaniv and Yossi Tamir

DISCUSSION PAPER 19

JERUSALEM, JULY 1977

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I. INTRODUCTION

In his important paper on the optimal linear income-tax, Eytan Sheshinski [3] proves that if the supply of labor in the economy is a non-decreasing function of the net wage rate, then the optimal tax schedule provides a positive lump-sum at zero income, and a marginal tax rate which is bounded above by a fraction that decreases with the minimum elasticity of the labor supply.

In his treatment of zero income, however, Sheshinski does not distinguish between individuals who in fact prefer no income, thus voluntarily avoiding the labor market, and individuals who are anxious to work but compelled to receive zero income as the inevitable result of unemployment. Focusing on this distinction, the purpose of this note is to extend Sheshinski's linear income-tax framework to take account of unemployment insurance by assuring each unemployed individual a compensation which is a linear function of his loss of income.

Using the states of the world approach to decision-making under uncertainty, individuals are assumed to determine their optimal labor supply by maximizing expected utility from the probabilistic states of employment and unemployment.[1] The sensitivity of the individual's labor supply to various parameters is examined, following Sheshinski, so as to provide the social planner with adequate information for the determination of the optimal tax-compensation schedule. This is done by maximizing expected social welfare subject to an actuarially fair constraint of zero expected net returns.

Assuming that leisure is a normal good and that the supply of labor in the economy is a non-decreasing function of the net wage rate in each possible state, it is seen that Sheshinski's result of a positive lump-sum at zero income still holds for individuals who voluntarily avoid the labor market. The optimal marginal tax rate imposed in a state of employment and the optimal marginal compensation rate given in a state of unemployment are each found to be bounded between terms that increase in value with the other rate and the odds for its occurence. Moreover, the optimal marginal tax rate is bounded above by the minimum of two terms, each of which is a decreasing function of the lowest labor supply elasticity with respect to one of the possible net wage rates, whereas the optimal marginal compensation rate is bounded below by the maximum of two terms,each of which is an increasing function of one of the lowest labor supply elasticities. A diagrammatical exposition of the efficient set from which the optimal combinations of the tax-compensation rates should be chosen is provided in Figure 1.

II. THE FORMAL STRUCTURE

Individuals are assumed to have an identical utility function, u, that depends on consumption, c, and labor, ℓ:

(1) $$u = u(c,\ell)$$

where u is continuously differentiable, strictly concave, with a positive marginal utility for consumption and a negative marginal utility for labor

(2) $$u_1 > 0, \quad u_2 < 0, \quad u_{11} < 0, \quad u_{22} < 0$$

Jobs are assumed to be provided through an official labor exchange where individuals offer their preferred amount of labor services. The labor exchange has a probability of 1-p of finding each individual a suitable job at a wage rate which is positively related to his innate ability, denoted by an index-number $n, (0 \leqq n \leqq \infty)$. Assuming, for simplicity, that this relation is linear, the employed individual's earnings, y, will be given by

(3) $$y = n \cdot \ell$$

With a probability of p, however, the labor exchange will fail to provide the individual with an adequate job, thus confronting him with an involuntary state of unemployment.

Let $T^i(y)$ be a linear income-tax function defined on y, such that

(4) $$T^i(y) = \begin{cases} -\alpha + (1-\beta)y & \text{for } i = em \\ -\alpha - \delta y & \text{for } i = ue \end{cases}$$

where *em* and *ue* denote the states of employment and unemployment respectively, and α, β and δ are the tax schedule parameters. α is a lump-sum tax ($\alpha < 0$) or subsidy ($\alpha > 0$) given to an individual who voluntarily avoids the labor market (offers $\ell = 0$). $(1-\beta)$ is the marginal tax rate on actual earnings when employed, while δ is the marginal compensation rate on loss of earnings when unemployed.[2]

Consumption is equal to after-tax earnings. That is,

(5) $$c^{em} = y - T^{em}(y) = \alpha + \beta y$$

in case of employment, and

(6) $$c^{ue} = - T^{ue}(y) = \alpha + \delta y$$

in case of unemployment.

III. INDIVIDUAL BEHAVIOR

Each individual chooses ℓ^* so as to maximize his expected utility

(7) $$Eu = (1-p)u(c^{em},\ell) + pu(c^{ue},0)$$

subject to his income constraint (3) and consumption constraints (5) and (6). Substituting (3), (5) and (6) into (7), the individual's problem becomes

(8) $$\max_{\ell} Eu = (1-p)u(\alpha + \beta n\ell,\ell) + pu(\alpha + \delta n\ell,0)$$

yielding as a first order condition for a maximum

(9) $$\frac{\partial Eu}{\partial \ell} = (1-p)(\beta n u_1^{em} + u_2^{em}) + p u_1^{ue} \delta n = 0$$

for all $n \geqq 0$.

The second order condition, which is derived by a differentiation of (9) with respect to ℓ, is assumed to hold at this point. That is,

$$\text{(10)} \qquad \frac{\partial^2 Eu}{\partial \ell^2} \equiv D = (1-p)\left[(\beta n)^2 u_{11}^{em} + 2\beta n u_{12}^{em} + u_{22}^{em}\right] + p(\delta n)^2 u_{11}^{ue} < 0$$

Equation (9)defines implicitly the optimal labor supply as functions of βn, δn, α and p

$$\text{(11)} \qquad \ell^* = \ell(\beta n, \delta n, \alpha, p)$$

Leisure is assumed to be a normal good. That is, by a differentiation of the first-order condition with respect to α, normality requires

$$\text{(12)} \qquad \frac{\partial \ell^*}{\partial \alpha} = -\frac{(1-p)(\beta n u_{11}^{em} + u_{12}^{em}) + p\delta n u_{11}^{ue}}{D} \lesseqgtr 0$$

It is also assumed that the supply of labor is a non-decreasing function of the net wage rate in each possible state. That is:

$$\text{(13)} \qquad \frac{\partial \ell^*}{\partial \beta} = -\frac{(1-p)n}{D}\left[u_1^{em} + \ell^*(\beta n u_{11}^{em} + u_{12}^{em})\right] \geqq 0$$

and

$$\text{(14)} \qquad \frac{\partial \ell^*}{\partial \delta} = -\frac{pn}{D}\left[u_1^{ue} + \delta n \ell^* u_{11}^{ue}\right] \geqq 0$$

It is easy to verify that under assumptions (13) and (14) the supply of labor is also a non-decreasing function of n, since

(15) $$\frac{\partial \ell^*}{\partial n} = \frac{\beta}{n}\frac{\partial \ell^*}{\partial \beta} + \frac{\delta}{n}\frac{\partial \ell^*}{\partial \delta} \gtreqless 0$$

Hence y is strictly increasing in n

(16) $$\frac{\partial y}{\partial n} = \frac{\partial (n\ell^*)}{\partial n} = \ell^* + n\,\frac{\partial \ell^*}{\partial n} > 0$$

Expected utility can also be shown, using (9), to be strictly increasing in n

(17) $$\frac{\partial Eu}{\partial n} = \left[(1-p)u_1^{em}\beta + pu_1^{ue}\delta\right]\ell^* > 0$$

while expected marginal utility is seen, in view of (12) and (15), to be strictly decreasing in n

(18) $$\frac{\partial Eu_1}{\partial n} = \left[(1-p)u_{11}^{em}\beta + pu_{11}^{ue}\delta\right]\ell^* + \left[(1-p)(\beta n u_{11}^{em} + u_{12}^{em}) + p\delta n u_{11}^{ue}\right]\frac{\partial \ell^*}{\partial n} < 0$$

IV. THE SET OF EFFICIENT MARGINAL RATES

Let f(n) be the density function of ability, i.e. the ratio of the number of individuals with ability n to the total number of individuals, such that $\int_0^\infty f(n)dn = 1$.

The social welfare function, V, is assumed to be the sum of individual utilities. Normalizing for size, the expected social welfare is given by

(19) $$EV = (1-p)\int_0^\infty u(c^{em},\ell)f(n)dn + p\int_0^\infty u(c^{ue},0)f(n)dn$$

The tax authorities choose α, β and δ such as to maximize (19) subject to the actuarial constraint that expected total tax proceeds be equal to zero, i.e.

$$(20) \qquad (1-p) \int_0^\infty \left[- \alpha + (1-\beta)y\right] f(n)dn + p \int_0^\infty \left[- \alpha - \delta y\right] f(n)dn = 0$$

(20) implies alternatively that at each $n \geqq 0$ the proportion of employed to unemployed individuals is $\frac{1-p}{p}$; that is, unemployment is homogeneously distributed within the population.

The actuarial constraint can also be written as

$$(21) \qquad \alpha = \left[(1-p)(1-\beta) - p\delta\right] \int_0^\infty yf(n)dn$$

which emphasizes that α is not subject to uncertainty.

Denoting the shadow price of constraint (21) by q, the function to be maximized becomes

$$(22) \qquad W = \int_0^\infty \Big\{(1-p)u(\alpha + \beta n\ell, \ell) + pu(\alpha + \delta n\ell, 0) - q\left[\alpha - [(1-p)(1-\beta) - p\delta]n\ell\right]\Big\} f(n)dn$$

The first-order conditions for a maximum of W with respect to α, β and δ are obtained, using (9), as[3]

(23) $$\frac{\partial W}{\partial \alpha} = \int_0^\infty (Eu_1 - q)\ f(n)dn + q\ \left[(1-p)(1-\beta) - p\delta\right] \int_0^\infty n\ \frac{\partial \ell^*}{\partial \alpha}\ f(n)dn = 0$$

(24) $$\frac{\partial W}{\partial \beta} = (1-p)\int_0^\infty (u_1^{em} - q)n\ell^* f(n)dn + q\ \left[(1-p)(1-\beta) - p\delta\right] \int_0^\infty n\ \frac{\partial \ell^*}{\partial \beta}\ f(n)dn = 0$$

(25) $$\frac{\partial W}{\partial \delta} = p\int_0^\infty (u_1^{ue} - q)n\ell^* f(n)dn + q\left[(1-p)(1-\beta) - p\delta\right] \int_0^\infty n\ \frac{\partial \ell^*}{\partial \delta}\ f(n)dn = 0$$

while (24) together with (25) yield

(26) $$\frac{\partial W}{\partial \beta} + \frac{\partial W}{\partial \delta} = \int_0^\infty (Eu_1 - q)n\ell^* f(n)dn + q\left[(1-p)(1-\beta) - p\delta\right] \int_0^\infty n\left(\frac{\partial \ell^*}{\partial \beta} + \frac{\partial \ell^*}{\partial \delta}\right)f(n)dn = 0$$

Following Sheshinski, it is obvious from (23) and (26) that $q \leqq 0$ is impossible in view of assumptions (12), (13) and (14). Sheshinski's proof that $\alpha^* \leqq 0$ is impossible holds in the present formulation as well in view of (15) and (18). Hence $q > 0$ and $\alpha^* > 0$, where the latter implies that $(1-p)(1-\beta) - p\delta > 0$ in view of constraint (21).

(24) and (25) can now be written as

(27) $$\frac{\partial W}{\partial \beta} = (1-p)\int_0^\infty u_1^{em} n\ell^* f(n)dn + q\int_0^\infty \left[\frac{(1-p)(1-\beta) - p\delta}{\beta}\ \frac{\beta}{\ell^*}\ \frac{\partial \ell^*}{\partial \beta} - (1-p)\right] n\ell^* f(n)dn = 0$$

(28) $$\frac{\partial W}{\partial \delta} = p\int_0^\infty u_1^{ue} n\ell^* f(n)dn + q\int_0^\infty\left[\frac{(1-p)(1-\beta)-p\delta}{\delta}\cdot\frac{\delta}{\ell^*}\frac{\partial \ell^*}{\partial \delta} - p\right]n\ell^* f(n)dn = 0$$

Since the first terms of (27) and (28) are positive, it follows from (27) that

(29) $$\left[(1-p) - \frac{(1-p)(1-\beta) - p\delta}{\beta}\lambda\right]\int_0^\infty n\ell^* f(n)dn \geqq \int_0^\infty\left[(1-p) - \frac{(1-p)(1-\beta) - p\delta}{\beta}\frac{\beta}{\ell^*}\frac{\partial \ell^*}{\partial \beta}\right]n\ell^* f(n)dn > 0$$

and from (28) that

(30) $$\left[p - \frac{(1-p)(1-\beta) - p\delta}{\delta}\eta\right]\int_0^\infty n\ell^* f(n)dn \geqq \int_0^\infty\left[p - \frac{(1-p)(1-\beta) - p\delta}{\delta}\frac{\delta}{\ell^*}\frac{\partial \ell^*}{\partial \delta}\right]n\ell^* f(n)dn > 0$$

where λ and η are defined to be the lowest elasticities of the labor supply function with respect to β and δ, respectively.

It is thus necessary from (29) that

(31) $$\beta^* > (1 - \frac{p}{1-p}\delta)\frac{\lambda}{1+\lambda} \qquad \text{for a given } \delta$$

and from (30) that

(32) $$\delta^* > (1-\beta)\frac{1-p}{p}\frac{\eta}{1+\eta} \qquad \text{for a given } \beta$$

Considering the actuarial constarint (21), and using (31) and (32), the optimal values of the marginal tax and compensation rates are found to be bounded within the following limits

$$(33) \qquad \frac{p}{1-p}\,\delta^* < 1-\beta^* < \min\left\{\frac{1}{1+\lambda} + \frac{\lambda}{1+\lambda}\,\frac{p}{1-p}\,\delta^*\ ,\ \frac{1+\eta}{\eta}\,\frac{p}{1-p}\,\delta^*\right\}$$

$$(34) \qquad \frac{1-p}{p}\,(1-\beta^*) > \delta^* > \max\left\{\frac{\eta}{1+\eta}\,\frac{1-p}{p}\,(1-\beta^*)\,,\ -\frac{1}{\lambda}\,\frac{1-p}{p} + \frac{1+\lambda}{\lambda}\,\frac{1-p}{p}\,(1-\beta^*)\right\}$$

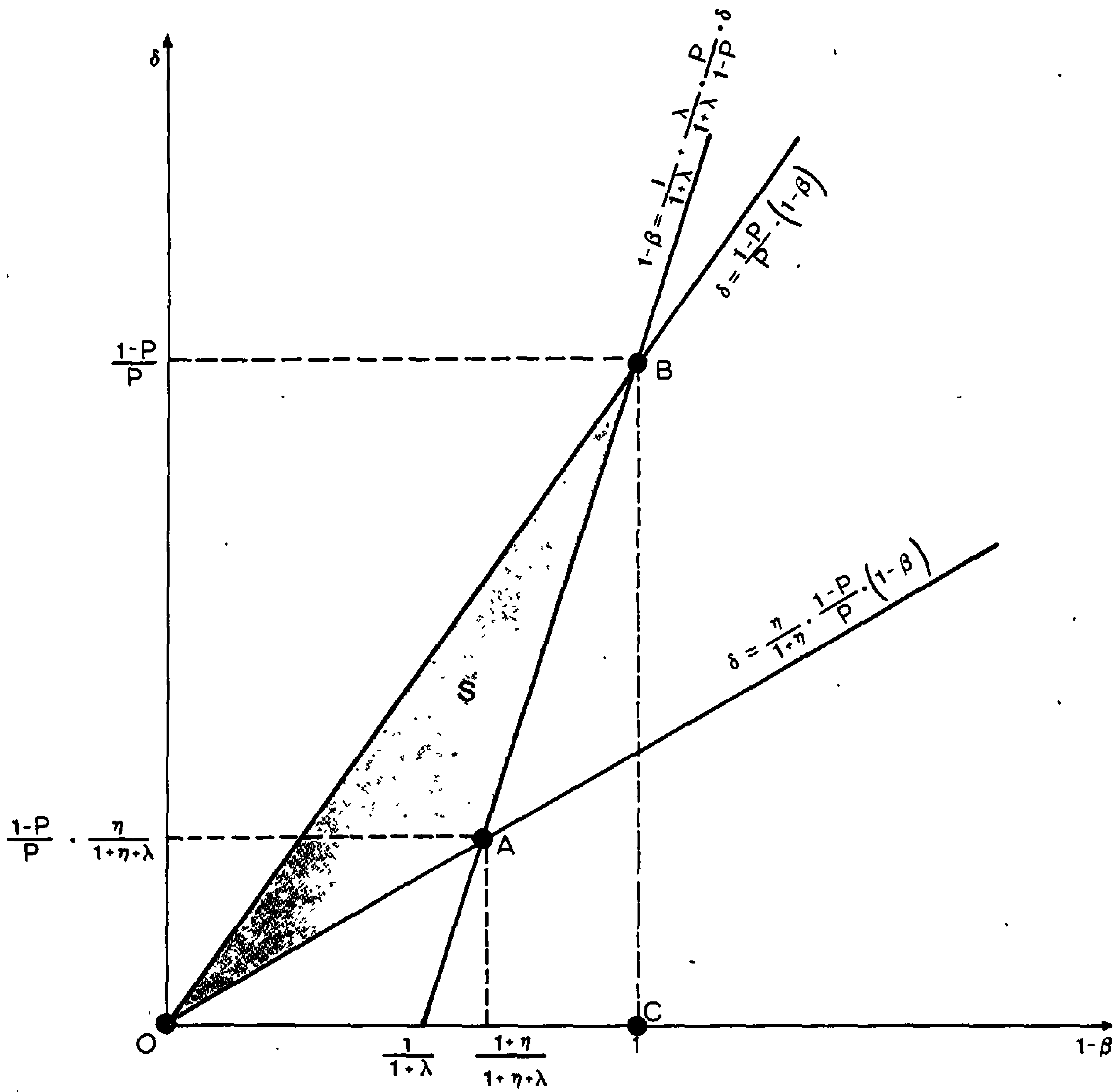
(33) and (34) define an efficient open set, S, out of which the optimal combinations of $1-\beta^*$ and δ^* are to be chosen. It is illustrated in Figure 1 by the area which is bounded by the triangle OAB, under the assumption that the minimum elasticities λ and η are independent of the tax-compensation rates. It is apparent that S is a decreasing function of λ and η and reaches a maximum for $\lambda = \eta = 0$, when it overlaps with the triangle OBC. However, when the value of λ or η depends on one of the decision variables, the exposition is less simple and requires the use of specific utility functions.

V. An Example

Consider the utility function $u(c,\ell) = c(L-\ell)$ where $L > 0$ is the maximum feasible labor supply. From the first-order condition (9) and the consumption constraints (5) and (6), the optimal labor supply is derived as

$$(35) \qquad \ell^* = \begin{cases} 0 & n \leqq n_0 \\ \dfrac{L[(1-p)\beta n + p\delta n] - (1-p)\alpha}{2(1-p)\beta n} & n > n_0 \end{cases}$$

FIGURE 1: Efficient Open Set (not including boundary points) of the Marginal Optimal Tax Rate ($1-\beta^*$) and Compensation Rate (δ^*) for Constant Minimum Elasticities of Labor Supply



where $n_0 = \frac{(1-p)\alpha}{L[(1-p)\beta + p\delta]}$. By (13) it is seen that $\frac{\partial \ell^*}{\partial \beta} = \frac{L-2\ell^*}{2\beta}$ which is non-negative for $\ell^* \leqq \frac{L}{2}$ or, using (35), for $n \leqq \bar{n} = \frac{(1-p)\alpha}{L[(1-p)\beta + p\delta] - (1-p)\beta}$, whereas by (14) one obatins $\frac{\partial \ell^*}{\partial \delta} = \frac{PL}{2(1-p)\beta} > 0$. Hence $\frac{\partial \ell^*}{\partial \beta} \frac{\beta}{\ell^*} = \frac{L-2\ell^*}{2\ell^*}$ and $\frac{\partial \ell^*}{\partial \delta} \frac{\delta}{\ell^*} = \frac{PL\delta}{2\ell^*(1-p)\beta}$, both of which decrease with ℓ^*. Let $\ell^* = \frac{L}{2}$ be the largest amount of labor supplied, so that $\frac{\partial \ell^*}{\partial \beta} \frac{\beta}{\ell^*} \geqq 0 = \lambda$ and $\frac{\partial \ell^*}{\partial \delta} \frac{\delta}{\ell^*} \geqq \frac{p\delta}{(1-p)\beta} = n$. Substituting into (33) and (34) yields

$$(33)' \qquad \frac{p}{1-p} \delta^* < 1-\beta^* < \min \left\{ 1 , \quad \frac{p\delta^* + (1-p)\beta^*}{1-p} \right\}$$

$$(34)' \qquad \frac{1-p}{p} (1-\beta^*) > \delta^* > \max \left\{ \frac{(1-p)(1-\beta^*)\delta^*}{p\delta^* +)1-p)\beta^*} , - \infty \right\}$$

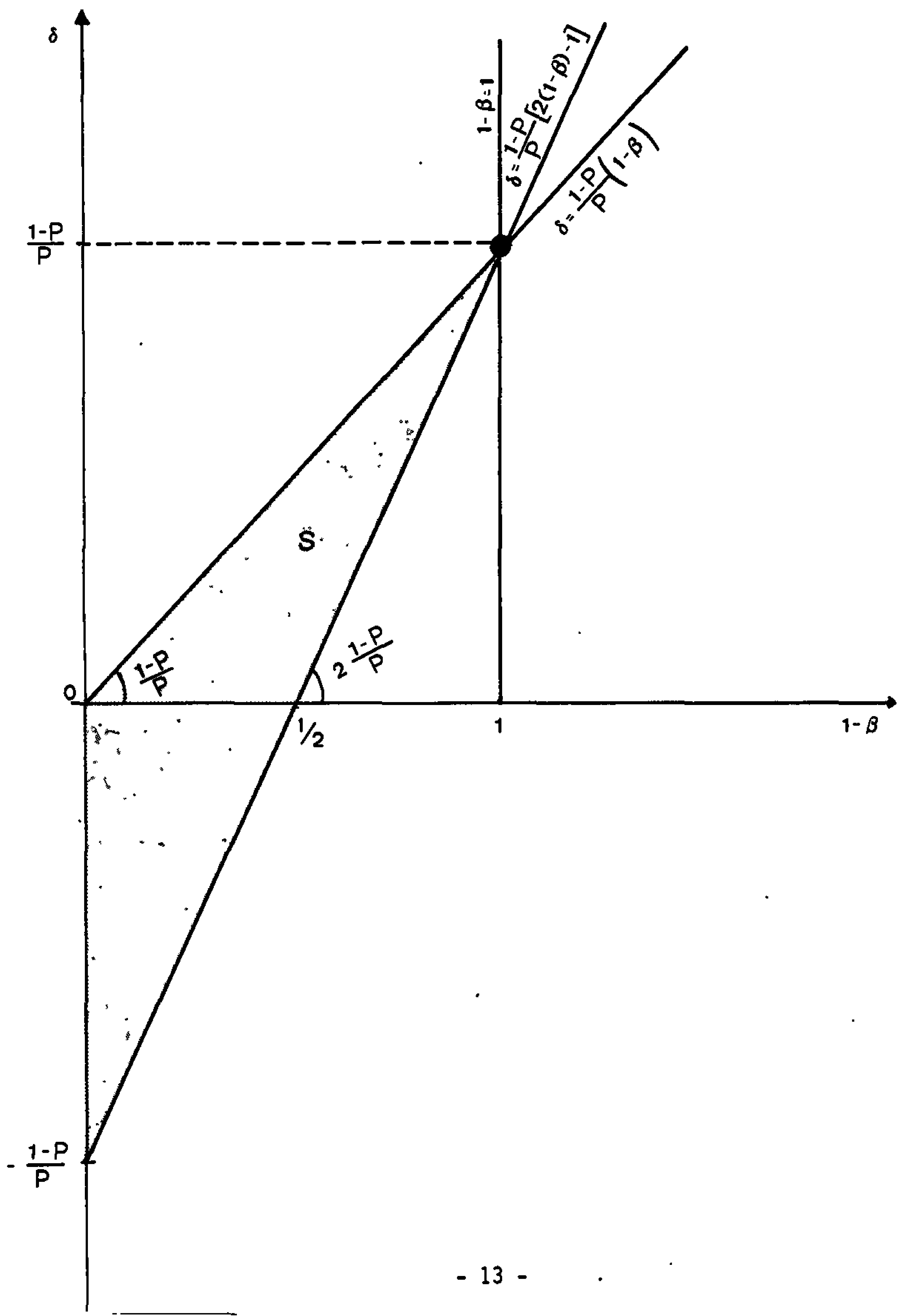
and after rearranging

$$(33)'' \qquad \frac{p}{1-p} \delta^* < 1-\beta^* < \frac{1}{2}(\frac{p}{1-p} \delta^* + 1)$$

$$(34)'' \qquad \frac{1-p}{p} (1-\beta^*) > \delta^* > \frac{1-p}{p} [2(1-\beta^*) - 1]$$

The efficient set of the optimal rates obtained in this case is illustrated in Figure 2. Note that in contrast to the case where the minimum elasticities are independent of the chosen rates, the present example allows negative optimal values of the compensation rate for sufficiently small values of the income tax rate.

FIGURE 2 Efficient Open Set of the Tax-Compensation Rates for the Utility Function $u=c(L-\ell)$



FOOTNOTES

(1) In a recent note, Sjoquist [2] uses the expected utility approach to point out some incorrect results obtained by Hartley and Revankar [1], who maximize utility of expected values to determine the individual's optimal labor supply under uncertainty resulting from possible unemployment. Both formulations, however, regard unemployment compensation as a lump-sum transfer. Variable compensation with respect to income loss has been recently introduced by Yaniv [4].

(2) A deliberate refusal of available jobs by an individual who has initially offered $\ell > 0$ can be made unattractive by rejecting his claim for compensation and imposing a penalty, for example, in the form of a reduction in α (demonstrated below to be positive) proportional to his initial offer.

(3) Note that for $p = 0$ equations (23), (24) and (25) are reduced to Sheshinski's two first order conditions, i.e.

$$\frac{\partial W}{\partial \alpha} = \int_0^\infty (u_1^{em} - q)f(n)dn + q(1-\beta)\int_0^\infty n \frac{\partial \ell^*}{\partial \alpha} f(n)dn = 0$$

$$\frac{\partial W}{\partial \beta} = \int_0^\infty (u_1^{em} - q)n\ell^* f(n)dn + q(1-\beta)\int_0^\infty n \frac{\partial \ell^*}{\partial \beta} f(n)dn = 0$$

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