TWO NOTES ON TAX EVASION

by

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I. A NOTE ON THE TAX EVADING FIRM

ABSTRACT

This note develops a general model of tax evasion applicable to any form of evasion that may be practiced by the firm. It shows that the firm's activity level is always separable from the evasion decision, that a tax rate increase must always decrease the firm's statement deviation from the true value of the magnitude subject to taxation, and that the amount of tax evaded can never increase; at most, when the firm acts as a withholding agent, it will remain unchanged. The results are applied to the case of payroll tax evasion.
I. INTRODUCTION

The tax evasion literature initiated by Allingham and Sandmo (1972), has mainly been concerned with the individual's decision to evade taxes through the underreporting of actual income. However, some attempts have recently been made to model the firm's decision to evade taxes, such as the evasion of sales taxes by underreporting actual revenue (Marrelli, 1984), the evasion of profit taxes by overstatement of actual production costs (Wang and Conant, 1988), or the evasion of withholding taxes by understating actual wage payments (Yaniv, 1988).

A common feature of these contributions is the conclusion that the firm's actual activity level is independent of tax evasion, as long as the latter is interiorly optimized. This finding is each time met with surprise, since models concerning the individual's tax evasion behavior fail to produce a symmetric separability between the labor-supply decision and the decision to evade taxes (e.g., Andersen, 1977, Baldréy, 1979, Pencavel, 1979). Additional similarities in the structure and comparative static implications of the firm's tax evasion models suggest that some of the interesting questions concerning the tax evading firm may be investigated with the aid of a single apparatus - regardless of the particular form of evasion.

This note proposes a general model of tax evasion applicable to any form of evasion that might be practiced by the firm. It shows that the firm's activity level is always separable from the evasion decision, and that if the firm decides on the unreported (or overreported) amount (rather than on its fraction of the misreported magnitude) - the alleged separability holds regardless of whether evasion is optimal or not. Moreover, it is shown that a tax rate increase must always decrease the firm's statement deviation from the true value of the magnitude subject to taxation, and that the amount of tax evaded can never increase; at most, when the firm acts as a withholding agent, it will remain unchanged. The note concludes with an application to payroll tax evasion.
II. THE MODEL

Consider a competitive firm facing a proportional tax rate, $\theta$, imposed on a certain tax base (revenue, wages, capital stock, profits, etc.) relating to its activity (production, employment, etc.). Denoting the firm's activity level by $A(\geq 0)$, its net profits if fully complying with the tax regulations, $\pi$, may be expressed as

$$\pi = \pi(A, \theta, m),$$

(1)

where $m$ represents a vector of market parameters. Clearly we assume that $\pi_{\geq 0}$, where $\pi_{\geq 0}$ holds when the firm acts as a withholding agent. The firm's optimal activity level, whether it aims at maximizing net profits or its utility of net profits, will be determined at $\pi_{A=0}$, given that $\pi_{AA}<0$.

Suppose, however, that the firm considers the possibility of evading part (or all) of its tax liability (or of another party's tax withholdings) by understating the true value of the tax base, or by overstating the true value of a certain economic magnitude which is deductible from the tax base. Denoting the firm's statement deviation from the true value by $S(\geq 0)$, its net profits if not detected, $\Gamma^{nd}$, will be

$$\Gamma^{nd} = \pi + \theta S.$$  

(2)

However, if its fraudulent statement is detected, the firm will have to pay back the evaded tax, $\theta S$, as well as to pay a penalty ('surcharge') which is a multiple $\delta>0$ of the amount evaded. Its net profits if detected, $\Gamma^d$, will thus be

$$\Gamma^d = \pi - \theta \delta S.$$  

(3)

Suppose furthermore that the firm is risk-averse, that its utility function, $U$, is defined on net profits only, and that the firm chooses $S^*$ and $A^*$ so as to maximize the expected utility of its prospect, $EU(\Gamma) = (1-p)U(\Gamma^{nd}) + pU(\Gamma^d)$, where $p$ denotes the (exogenously given) probability
of detection. The first-order conditions for an interior maximum will be

$$EU = \Theta((1-p)U'(\Gamma^d) - \delta p U'(\Gamma^d)) = 0,$$

(4)

implying that tax evasion will be practiced ($S^* > 0$) if $p(1+\delta) < 1$, and

$$EU_A = \pi_A EU'(\Gamma) = 0.$$

(5)

Since $EU'(\Gamma) > 0$, eq. (5) reduces to $\pi_A = 0$. Hence, the activity decision is independent of the firm's attempt to evade taxes by fraudulent misreporting - regardless of whether misreporting is optimal or not. \(^3\) Marrelli (1984, p. 187) and Wang and Conant (1988, p. 581) concluded, however, that tax evasion has no influence on the output decision only if the firm "is able to equate the marginal rate of substitution (between the alternative profit levels) to the real price of evasion". This restriction arises from their assumption that the firm decides on the fraction of its statement deviation from the actual value, rather than on the absolute amount. To see this, define $S = sf(A)$, where $f(A)$ is the true value of the misreported magnitude and $s$ is the deviation rate. Eq. (5) then becomes

$$EU'(\Gamma)\pi_A + sf'(A)\Theta((1-p)U'(\Gamma^d) - \delta p U'(\Gamma^d)) = 0,$$

(5')

which reduces to $\pi_A = 0$ only after the substitution of (4), and regardless of the sign of $f'(A)$. \(^3\)

Consider now the implications of tax rate changes on the evading firm. To begin with, notice that by Hotteling's lemma

$$\frac{d\pi}{d\theta} = \frac{d\pi_A}{d\theta} + \pi_e = \pi_e.$$

(6)

That is, the total effect on legitimate net profits of a tax rate change which affects the optimal activity level - reduces to its direct effect only. Since the random components of the evading firm's profits are independent of actual employment, its evasion response to a tax rate change
may be derived (from (4)) as if its activity level were predetermined. Totally differentiating (4) with respect to $S$ and $\theta$ we thus obtain

$$\frac{dS^*}{d\theta} = \frac{[R_A(\Gamma^d) - R_A(\Gamma^{nd})]\pi_e}{\theta[\delta R_A(\Gamma^d) + R_A(\Gamma^{nd})]} - S$$

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where $R_A(\Gamma) = -U''(\Gamma)/U'(\Gamma) > 0$ is the Arrow-Pratt absolute risk-aversion measure. Under the accepted assumption of decreasing absolute risk-aversion [$R_A(\Gamma^d) > R_A(\Gamma^{nd})$], the sign of (7) is unambiguously negative. This, at first glance, is not surprising; since the penalty is imposed on the evaded tax, an increase in the firm's tax rate would discourage misreporting, in the same way as an increase in the income tax rate discourages underreporting in Yitzhaki's (1974) well-known formulation of the tax evasion problem. However, the sign of (7) is also negative for $\pi_e = 0$, regardless of risk-aversion behavior, and in spite of the fact that a higher tax rate increases net profits in case of non-detection. Evidently, the fall in net profits in case of detection more than offsets the former effect, inducing truth-telling. Moreover, when $\pi_e = 0$, eq. (7) reduces to a very simple term, $-S/\theta$, implying that the elasticity of $S^*$ with respect to $\theta$ is unity. In other words, a tax rate increase would not affect the amount of tax evaded. The employer, in fact, would hand the increased amount withheld over to the tax collector.

However, when $\pi_e < 0$, it is not immediately clear what happens to the amount of tax escaping the tax collector. Appropriate differentiation reveals that

$$\frac{d(\Theta S^*)}{d\theta} = \frac{dS^*}{d\theta} = \frac{[R_A(\Gamma^d) - R_A(\Gamma^{nd})]\pi_e}{\delta R_A(\Gamma^d) + R_A(\Gamma^{nd})} \leq 0$$

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if $\pi_e \leq 0$, respectively. Thus, a tax rate increase would never be able to increase the amount evaded. At most, when $\pi_e = 0$, it would leave the amount evaded unchanged. Otherwise, the amount evaded would always decrease.\(^6\)
III. PAYROLL TAX EVASION: AN APPLICATION

Taxes on payrolls, levied in most countries to finance social security programs, are usually collected from both employees and employers, up to a certain ceiling per employee. The employees' share is withheld at the source by the employer, who handles the reporting of earnings and the remittance of taxes to the authorities. Hence, both the employees' and the employer's taxes can be evaded by underreporting actual taxable payroll.

Consider a competitive employer facing a fixed wage rate, \( w \), per worker employed over a given period of time, who is required by law to contribute to the social security agency a proportion \( \theta \) of his payroll, as well as to withhold and remit to the agency a proportion \( t \) of his employees' earnings as their own contribution. Suppose, for simplicity, that there is no ceiling on tax collection. The employer's net profits if fully complying with the law will then be

\[
\pi(L, \theta, w) = V(L) - (1+\theta)wL, \tag{1'}
\]

where \( L \) and \( V(L) \) represent his employment level and value of output, respectively. However, if the employer underreports his payroll by the amount \( S (swL) \), his net profits will be

\[
\Gamma^d = \pi + (\theta + t)S \tag{2'}
\]

if not detected, and

\[
\Gamma^d = \pi - \delta(\theta + t)S \tag{3'}
\]

if detected.7 Obviously, we may make use now of eq. (7) to predict that an increase in either \( \theta \) or \( t \) would unambiguously discourage underreporting (substituting \( \pi^e = -wL \) or \( \pi^e = 0 \) in (7), respectively), and of eq. (8) to determine that the amount of tax evaded, \( (\theta + t)S \), should fall as \( \theta \) increases, but must stay intact as \( t \) varies. It also follows that an increase in \( \theta \) which is accompanied by a "compensated" equal
percentage-point decrease in t (so as to keep the overall payroll tax rate, 0+t, unchanged) would necessarily reduce the amount evaded. Hence, the greater the employer's share in the overall tax liability, the smaller will be the amount of tax escaping the social security agency.\textsuperscript{a}
ENDNOTES

As always, subscripts are used to denote partial derivatives; that is, \( \pi_1 = \pi / \pi_1 \), \( \pi_{11} = \pi_{22} / \pi_1^2 \).

One may wonder why a similar separability does not arise in the individual's tax evasion analyses. The answer seems to be that the individual's utility is defined not only on net income but on the activity level as well. If, for example, the firm's utility function consisted of an additional element, \( \phi(A) \), its optimal activity level in the absence of evasion would be determined at \( \pi_A = -\phi(A)/U'(\pi) \). In the presence of evasion, the optimum condition would become \( \pi_A = -\phi(A)/EU'(\Gamma) \), which clearly depends on tax evasion.

The same restriction holds if, as assumed by Yaniv (1988), the firm decides on the amount to be reported, which implies that \( S \) (and thus the penalty) is a function of \( A \).

Notice that \( dS^*/dS = -EU_{se}/EU_{ss} \), where \( EU_{se} = \theta(1-p)U'(\Gamma_md) \{[\pi_e - \delta S]R_A(\Gamma_md) - (\pi_e + S)R_A(\Gamma_md) \} \) and \( EU_{ss} = -\theta^2(1-p)U'(\Gamma_md) \{\delta R_A(\Gamma_md) + R_A(\Gamma_md) \} \) is the second-order condition for the maximization of expected utility, expressed (using (4)) in terms of the risk-aversion measures.

Still, Marrelli (1984, P. 182) concluded that "it is not necessarily true that higher tax rates induce larger declarations, even if the fine is imposed on the evaded tax". The reason for this discrepancy is that, as mentioned earlier, Marrelli defined the firm's problem as that of deciding on the fraction of actual revenue to be declared. An increase in the excise tax rate reduces the quantity purchased by consumers - and thus actual revenue. If actual revenue falls more strongly than the undeclared amount, the undeclared proportion might, of course, increase. This problem does not arise in Wang and Conant (1988), since actual production (and thus actual cost) is independent of the profit tax rate. In any case, as shown by (7), the statement "higher tax rates induce larger declarations when the fine is imposed on the evaded tax" is always true if the firm decides on the amount of under declaración.
The effect of a tax rate increase on the amount of tax evaded is usually not investigated in tax evasion models, apparently due to the intuitive feeling that the result is bound to be ambiguous. An exception is Christiansen (1980), who derived a negative relationship using a modified version of Yitzhaki's (1974) model, where evaded taxes are viewed as the individual's decision variable. Eq. (8) implies, however, that Christiansen's result may also be derived directly from Yitzhaki's model: defining \( \pi = (1-\theta)W \) to be net income in the absence of evasion, where \( W \) denotes actual income, we have \( \pi_e = -W \). Substituting into (8) we obtain the negative relationship in terms of Yitzhaki's formulation, where \( S = W - X \) (actual income minus reported income), and \( \Gamma^d \) and \( \Gamma^{nd} \) interpreted as net income in case of detection and non-detection, respectively.

We ignore here the possibility that detection of payroll tax evasion may also be the result of workers' exposure to a contingency covered by social security schemes, in which case the employer might be obliged to bear the cost of compensating workers whose insurance contributions were evaded.

The effect on underreporting and the amount evaded of a "compensated" increase in \( \theta \) may also be derived directly from (7) by simply dropping its second term (since \( \theta + t \) are held constant). Thus

\[
\frac{dS^*}{d\theta} = \frac{[R_A(\Gamma^d) - R_A(\Gamma^{nd})]WL}{(\theta + t)[\partial R_A(\Gamma^d) + R_A(\Gamma^{nd})]} < 0. \tag{7'}
\]

Since underreporting falls while the overall tax rate does not change, it follows that the amount evaded decreases.
REFERENCES


II. A NOTE ON THE TAX EVADING INDIVIDUAL

ABSTRACT

This note demonstrates that the ambiguity embodied in Allingham and Sandmo's (1972) model with regard to the effect of a change in the income tax rate on tax evasion is dissolved in the intuitively expected way if (a) the worst that can happen to a detected evader is the confiscation of his entire undeclared income, and (b) the relative-risk aversion is constant and bounded from above by the inverse of the penalty rate. The empirical relevance of these restrictions is broadly discussed.
I. INTRODUCTION

In their seminal paper on income tax evasion, Allingham and Sandmo (1972, hereafter AS) concluded that under decreasing absolute risk-aversion the effect of an increase in the income tax rate on declared income is ambiguous, since the negative substitution effect of a tax increase is opposed by a positive income effect. In an important note on AS's paper, Yitzhaki (1974) argued that if the penalty on tax evasion is proportional to the evaded tax, rather than to the undeclared income (as assumed by AS), an increase in the income tax rate must increase declared income because the substitution effect is eliminated. While Yitzhaki's modification of the AS's model generates an unambiguous result, it is, quite oddly, of a counter-intuitive nature. Over the two decades that followed, economists have often expressed disappointment with the failure of the theoretical framework to predict that declared income is negatively related to the income tax rate, a result which accords much more with common sense and intuition, although still lacking a strong empirical support.

The purpose of the present note is to show, under empirically verified restrictions on the penalty scheme and risk-aversion behavior, that the desired prediction has been here all the time. More specifically, an unambiguous negative relationship between declared income and the income tax rate is actually embodied in the AS's model if (a) the worst that can happen to a detected evader is the confiscation of his entire undeclared income, and (b) the relative risk-aversion is constant and bounded from above by the inverse of the penalty rate. Section II employs AS's model to derive this proposition. Section III discusses the empirical relevance of the imposed restrictions, and concludes with some related remarks.
II. TAX EVASION AND THE INCOME TAX RATE UNDER CONSTANT RELATIVE RISK-AVERSION

Consider the AS model where a risk-averse taxpayer is allowed to declare less than his actual income, \( W \). Declared income, \( X \), is taxed by a constant rate, \( \theta \), whereas undeclared income, \( W-X \), is taxed, if detected, by a higher rate, \( \pi \). The taxpayer chooses \( X \) so as to maximize his expected utility

\[ E[U] = (1-p)U(Y) + pU(Z), \]  

(1)

where \( p \) is the (exogenously given) probability of detection, and

\[ Y = W - \theta X \]  

(2)

\[ Z = W - \theta X - \pi(W-X), \]  

(3)

represent his net income in case of detection and non-detection, respectively.

The first-order condition for the maximization of (1) is

\[ \frac{dE[U]}{dX} = -\theta(1-p)U'(Y) + (\pi-\theta)pU'(Z) = 0, \]  

(4)

from which the taxpayer's response to a change in \( \theta \) may be derived. This is given by

\[ \frac{dX}{d\theta} = \frac{\pi}{D} \]  

(5)

where \( D = \theta^2(1-p)U''(Y) + (\pi-\theta)^2pU''(Z) < 0 \) is the second-order condition for the maximization of (1) and \( R_A(I) = -U''(I)/U'(I) > 0 \) is the Arrow-Pratt absolute risk-aversion measure, evaluated at \( I=Y,Z \).
Under decreasing absolute risk-aversion \([R_a(Z) > R_a(Y)]\), the sign of (5) is ambiguous, as asserted by AS. However, given that the relative risk-aversion is constant \([R_a(I)I = c]\), (5) implies that

\[
\frac{dX^*}{d\theta} > \frac{Y - Z}{\pi} = 0 \text{ if } c < \frac{Y Z}{\pi - \theta},
\]

or, by substituting (2) and (3) into (6) and rearranging, that

\[
\frac{dX^*}{d\theta} > \frac{W}{\alpha (1 + \beta)} = 0 \text{ if } c < \frac{W X}{\alpha (1 + \beta)},
\]

where \(\alpha = (W - \theta X)/(W - X)\) and \(\beta = (1 - \pi)/(\pi - \theta)\). Clearly, \(\alpha > 1\) and \(\beta \leq 0\) if \(\pi \leq 1\). Hence, \(\pi \leq 1\) ensures that \(\alpha (1 + \beta) > 1\), so that \(dX^*/d\theta < 0\) if \(c \leq 1\). However, since \(\beta\) varies inversely with \(\pi\), the stricter the restriction imposed on \(\pi\), the higher the ceiling on \(c\) allowed for the satisfaction of \(dX^*/d\theta < 0\) (recall also AS's result that \(dX^*/d\pi > 0\), so that \(W/X\) rises as \(\pi\) falls). Still, since all that is known on \(W/X\) is that it exceeds unity, the upper bound on \(c\) which may safely be identified as yielding the desired prediction is just \(1 + \beta = (1 - \theta)/(\pi - \theta)\), which rises with \(\theta\). Referring again to its minimal value, we conclude that if \(\pi \leq 1\) and \(c \leq 1/\pi\), condition (7) surely implies that when the fruits of evasion become sweeter, a rational taxpayer will take a bigger bite.

III. EMPIRICAL RELEVANCE AND RELATED REMARKS

Summarizing almost two decades of theoretical research on portfolio choice under uncertainty, James Tobin (1969) wrote "...it is very difficult to derive propositions that are simultaneously interesting and general. In particular, the Neumann-Morgenstern hypothesis of utility maximization will not, unaided, tell us much about portfolio choices. To get propositions with significantly more content than the prescription that the investor should maximize expected utility, it is necessary to place restrictions on
his utility function or his subjective probability estimates..." (p. 13). Following Tobin's advise, this note has reemployed the AS tax evasion model to examine the relationship between tax evasion and the income tax rate under the restriction that the relative risk-aversion is constant."

The note's results suggest that if the penalty rate is 100% (i.e., the entire undeclared income is confiscated), a negative relationship between declared income and the income tax rate will unambiguously hold if the relative risk-aversion coefficient does not exceed unity. In more realistic cases, where the penalty rate is lower, the desired prediction will also hold for a higher than unity coefficient of relative risk-aversion. In the U.S., a detected evader is obliged to pay less than 1.5 times his *evaded taxes* for most violations (Pencavel, 1979, p. 122), whereas the ratio of tax payments to adjusted gross income averages less than 20% (ibid. p. 122). This means that the penalty rate on undeclared income is less than 30%. Hence, if the penalty scheme in the U.S. were made dependent on undeclared income, keeping the punishment level intact, a negative relationship between declared income and the income tax rate would be predicted for a relative risk-aversion coefficient that does not exceed 3.33.

Economists have employed cross-sectional data on household assets to establish properties of households' utility functions. In a frequently cited work, Friend and Blume (1975) found, using data from the Survey of Financial Characteristics of Consumers (SFCC), that constant relative risk-aversion is a fairly accurate description of investor behavior: "if there is any tendency for increasing or decreasing proportional risk aversion, the tendency is so slight that for many purposes the assumption of constant proportional risk aversion is not a bad first approximation" (p. 915). This seems to support Arrow's (1965) assertion that "broadly speaking, the relative risk-aversion must hover around one" (p. 37), which implies that the utility function is logarithmic. However, Friend and Blume, estimating the market price of risk to determine the coefficient of relative risk-aversion for the typical household, argued that this coefficient is greater than one and may be as high as two. This still conforms with our above finding of the upper bound on the relative

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risk-aversion coefficient under which evasion will unambiguously increase with the income tax rate.

Several authors have extended the basic AS's model to take account of the possible interactions between tax evasion and the labor supply decision, concluding that unambiguous comparative static predictions cannot be derived out of a general formulation of the utility function (e.g., Andersen, 1977; Baldry, 1979; Pencavel, 1979; Cowell, 1985). In one of these contributions, Isachsen and Strom (1980) consider the more realistic case where individuals divide their working time between the regular and the irregular economy, declaring all their earnings from the former, but none of their earnings from the latter. After discussing the entry conditions and corner solutions, Isachsen and Strom turn to examine the individual's inner allocation problem under the assumption that the utility function is logarithmic in income and leisure. This assumption implies that total working time is fixed, so that the model becomes quite similar to that of AS, with 'black' earnings playing the role of undeclared income and 'official' earnings being declared income. The most interesting prediction arising under the logarithmic utility function is that an increase in the income tax rate would push more labor into the irregular economy. However, Isachsen and Strom were only able to show this result for the very specific case of \( \pi=1 \) (i.e., only if all 'black' earnings are confiscated if caught). The present note derives an analogous implication directly from the AS model, and under much less restrictive assumptions on the penalty scheme and risk-aversion behavior.
FOOTNOTES

'It is interesting to note, however, that when the individual's income taxes are withheld by the employer, who may then choose to remit only part of them to the government by underreporting his actual wage payments, both the substitution and income effects of a tax rate increase act, as expected, to decrease declaration, given that the penalty is imposed on undeclared payments (Yaniv, 1988).

The most relevant study is that of Clotfelter (1983), who, using actual individual tax returns, concluded that higher tax rates tend to stimulate evasion. Cox (1984) and Slemrod (1985) have questioned this finding, claiming to find no evidence of a negative effect of marginal tax rates on compliance.

The R.H.S. term of (5), which represents the negative substitution effect of a change in $\theta$, looks different that that obtained by AS, but is actually identical to AS's term, \((1/D)[(1-p)U'(Y) + pU'(Z)]\). It is obtained by substitution (4) into AS's term, a step taken by AS to rewrite the income effect only.

Notice that \(c > 1\) does not necessarily imply that \(dx*/d\theta > 0\).

It is interesting to note that if the taxpayer's income stems from labor only, AS's assumption that actual income is constant is actually equivalent to assuming that the relative risk-aversion is constant (which implies that labor supply is fixed).

More recently, Morin and Suarez (1983) found the coefficient of relative risk-aversion to be slightly decreasing for wealth levels up to $100,000, after which it becomes approximately constant. However, when restricting the sample and the wealth definitions to approximate those used by Friend and Blume, Morin and Suarez found that the risk-aversion results are modified to resemble the patterns identified by Friend and Blume.
The logarithmic utility function, already suggested by Bernoulli, is frequently used in finance contexts (e.g., Kraus and Litzenberger, 1975, Rubinstein, 1977), and in empirical applications analyzing von Neuman-Morgenstern utility functions (Viscusi and Evans, 1990).

Brookshire, Thayer, Tschirhart and Schulze (1985) used, for example, this last result (choosing \( U(I) = -I^{-1} \), for which the coefficient is two) in order to determine an upper bound on the ratio of marginal utilities from income in the states of earthquake and no earthquake, a parameter necessary to solve for the price difference between houses in and out of unsafe areas.
REFERENCES


