THE NATIONAL INSURANCE INSTITUTE
Research and Planning Administration

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Discussion Paper **No. 61**

Jerusalem, Israel, December 1995

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WAGE BILL UNDERREPORTING AND THE WITHHOLDING TAX RATE: A TAX EVASION PARADOX

Despite the growing evidence on tax noncompliance by small corporations,[1] corporate evasion of tax withholdings has received very little attention in the tax evasion literature. In a seminal discussion of this issue, I considered (Yaniv, 1988) a competitive employer, facing a fixed wage rate, w, per hour of employed labor, N, who is required by tax regulations to withhold a given percentage, t, of his workers' wages, wN. The employer, however, may decide to remit to the tax collector, via understating his actual wage bill, less than the amount withheld.[2] The analysis shows, inter alia, that the employment decision is independent of tax evasion (as well as of the withholding tax rate) and that the effect on declared wages, Z ($<wN$), of an increase in the withholding tax rate is captured by a very simple term: $dZ/dt = (wN-Z)/(t-\theta)$, where θ denotes the profit tax rate (see Appendix).[3] Consequently, $d[(t-\theta)(wN-Z)]/dt = 0$, so that a tax rate increase would not affect the amount of tax evaded. A similar result has

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[1]See Rice (1992) for a discussion of the corporate tax gap (i.e., the difference between tax receipts based on voluntary reporting and what the IRS views as the correct amount of tax due).

[2]The employer may also collaborate with his employees in withholding less than required [Yaniv (1992), Baldry (1993)].

[3]Since understating actual wages means overstating actual profits, a prerequisite for such behavior is, of course, that the withholding tax rate exceeds the profit tax rate (i.e., $t-\theta > 0$), a condition which is quite likely to hold in practice: while the effective tax rates on profits often lie drastically below the statutory rates due to generous tax credits and depreciation allowances, the withholding tax rates are usually not subjected to most of the deductions allowed to employees upon filing a tax return.

been obtained in my recent note (Yaniv, 1995), derived as a specific case of a general model of tax evasion applicable to any type of tax that might be evaded by the firm.

This result bears, however, a paradoxial implication for the employer's behavior, left so far unnoticed: since the amount of tax withheld necessarily increases, it follows that the amount of taxes paid increases by exactly the same magnitude.[4] Being aware of that, the tax collector can easily calculate the difference between the amount of taxes paid before and after the tax change, divide it by the increase in the tax rate, and get to know the actual wage bill! Hence, the first-best response of the tax-evading employer to a tax rate increase is self-defeating, thus the *least* desirable.[5]

How then should the employer react to the tax change? Obviously, he must see to it that his declaration fully coincides with the tax collector's evaluation of the wage bill on the basis of this declaration. The only possible solution to this problem is to declare the same level of wages as before.[6] The tax collector, calculating the difference per tax rate change between the amount of taxes paid before and after the tax change, would then obtain

[4]As picturesquely phrased by Hagedorn (1989), the employer in fact hands the increased amount withheld over to the tax collector. Mathematically, $d[\theta V(N) + (t-\theta)Z]/dt = Z + (t-\theta)(wN-Z)/(t-\theta) = wN = d(twN)/dt$, where $V(N)$ denotes the value of output, assumed to be accurately declared (if easily detectable).

[5]This paradox has been pointed out by an anonymous student in my Economics of Crime course at Tel-Aviv University.

[6]That is, the problem is to find the value of dZ/dt for which $d[\theta V(N) + (t-\theta)Z]/dt = Z + (t-\theta)(dZ/dt) = Z$. This requires setting $dZ/dt = 0$. Notice that declaring the actual wage bill (the first intuitive response to avoiding a certain detection) is not a desirable solution because the amount of taxes paid would then increase by more than the amount withheld, thus the tax collector calculation would still yield a greater wage bill than declared (the difference contributed by past underdeclaration, which is bound to be disclosed).

exactly the figure declared. Of course, the tax collector might suspect that the employer (being aware of the authorities' ability to figure out his actual wage bill from his own declaration) has deliberately deviated from his first-best move, but to find this out the tax collector must incur investigation costs, as is the case with any other taxpayer.

Leaving the level of declaration intact implies that the employer *increases* the amount of taxes paid by *less* than the additional amount withheld, *increasing* as well the amount of tax evaded. In my recent note on the (any) tax-evading firm, I concluded, however, that a tax rate increase can never increase the amount of tax evaded by the firm; at most, when the firm acts as a withholding agent, the amount evaded will remain unchanged (otherwise it always decreases). This conclusion ignores, as it is now apparent, the firm's assessment of whether its tax change adjustment embodies any contradicting (thus self-implicating) information. While an expected decrease in the amount evaded cannot disclose such information,[7] an expected constancy can. The skillful evader of tax withholdings, faced with a tax rate increase, would thus attempt to obscure his tax evasion *by evading even further* - a second-best paradoxial reaction to a first-best paradox.

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[7]This is so since the amount of taxes paid would then be expected (by the tax collector) to increase more than the firm's tax liability. Mathematically, denoting the firm's tax base by B and the amount declared by D, $d[t(B-d)]/dt < 0$ if $dD/dt > (B-D)/t$, which is exactly the condition for $d(tD)/dt > d(tB)/dt$. Consequently, dividing the additional amount paid by the tax rate change would yield a magnitude considered to be greater than the tax base, revealing nothing on the relation between the tax base and the amount declared.

APPENDIX

The employer's problem is to choose N and Z which maximize his expected utility, $EU = (1-p)U(\pi^{nd}) + pU(\pi^{d})$, subject to

$$\pi^{nd} = (1-\theta)[V(N) - wN] + (t-\theta)(wN-Z) \tag{1}$$

$$\pi^{d} = (1-\theta)[V(N) - wN] - (\delta-1)(t-\theta)(wN-Z), \tag{2}$$

where V(N) denotes the value of output, p - the probability of detection, $\delta>1$ - the penalty rate on evaded taxes, and π^{d}, π^{nd} - net profits in case of detection and non-detection, respectively.

The first-order conditions for an interior maximum are

$$\frac{d(EU)}{dN} = \{(1-\theta)[V'(N) - w] + (t-\theta)w\}(1-p)U'(\pi^{nd}) + \{(1-\theta)[V'(N) - w] - (\delta-1)(t-\theta)w\}pU'(\pi^{d}) = 0 \tag{3}$$

$$\frac{d(EU)}{dZ} = - (t-\theta)[(1-p)U'(\pi^{nd}) - (\delta-1)pU'(\pi^{d})] = 0. \tag{4}$$

Substituting (4) into (3) yields $V'(N) - w = 0$, so that the employment decision is independent of tax evasion at the optimum (as well as of the withholding tax rate). Totally differentiating (4) with respect to t yields

$$\frac{dZ}{dt} = \frac{(t-\theta)(wN-Z)[(1-p)U''(\pi^{nd}) + (\delta-1)^2pU''(\pi^{d})]}{(t-\theta)^2[(1-p)U''(\pi^{nd}) + (\delta-1)^2pU''(\pi^{d})]} = \frac{wN-Z}{t-\theta}. \tag{5}$$

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TAX EVASION AND MONOPOLY OUTPUT DECISIONS

In a recent article, Wang (1990) has reexamined the monopoly output decision when it opts to evade profit taxes by overstating its production costs. Contrary to Wang and Conant (1988), who examine this issue under the assumption that the probability of detection and the penalty rate are exogenously fixed, Wang generalizes the model by endogenizing these variables, allowing both to increase with the overstated amount. Wang concludes that under such modifications the monopolist's output decision is no longer separable from the evasion decision, as shown in Wang and Conant. Consequently, the conventional view that profit taxes are neutral with respect to the monopolist's profit-maximizing output level no longer holds; the tax-evadin monopolist may even restrict output, deepening monopoly distortion.

This note shows that Wang's inseparability conclusion is, regrettably, incorrect. Quite surprisingly, the neutrality of profit taxes is preserved even under his more realistic assumptions. Moreover, when the monopolist decides on the overstated *amount* rather than on the *proportion* of cost overstatement, as assumed by Wang and by Wang and Conant, the separability of the output decision holds even if evasion is not optimally chosen. This is in contrast to Wang and Conant's conclusion that tax evasion has no influence on the output decision "only if the profit-tax-paying firm is able to equate the marginal rate of substitution (between the alternative profit levels) to the real price of evasion" (1988, p. 581). Consequently, separability holds even at a corner solution.

Following Wang, consider a monopolistic firm, the profit of which is subject to a proportional tax rate, t. Denoting the firm's output level by Q, its legitimate net profit is $(1-t)[R(Q) - C(Q)]$, where $R(Q)$ and $C(Q)$ represent total revenue and total production costs, respectively. Suppose, however, that the firm can evade part (or all) of its tax liability by probability of detection, p, and the penalty rate on evaded taxes, s (>1), increase with the amount of cost overstatement, $\delta C(Q)$. That is, $p = p[\delta C(Q)]$ and $s = s[\delta C(Q)]$, where $p'>0$ and $s'>0$. The momopolist's problem is to choose the output level and the cost overstatement fraction so as to maximize its expected utility

$$EU = \{1 - p[\delta C(Q)]\}U(\pi_1) + p[\delta C(Q)]U(\pi_2), \tag{1}$$

where $\pi_1 = (1-t)[R(Q) - C(Q)] + t\delta C(Q)$ (2)

and $\pi_2 = \pi_1 - s[\delta C(Q)]t\delta C(Q),$ (3)

denote net profit in case of non-detection and detection, respectively. The first-order conditions for an interior maximum of expected utility are[1]

$$\frac{\partial EU}{\partial Q} = (R' - C')(1-t)[(1-p)U'(\pi_1) + pU'(\pi_2)] + \{(1-p)tU'(\pi_1) + p(1-s-s'\delta C)tU'(\pi_2) - p'[U(\pi_1) - U(\pi_2)]\}\delta C' = 0 \tag{4}$$

and

$$\frac{\partial EU}{\partial \delta} = \{(1-p)tU'(\pi_1) + p(1-s-s'\delta C)tU'(\pi_2) - p'[U(\pi_1) - U(\pi_2)]\}C = 0. \tag{5}$$

Substituting (5) into (4), we immediately obtain $R'=C'$. Hence, the monopolist's output level is independent of its attempt to evade taxes by fraudulently overstating its production costs *even when the probability of detection and the penalty rate vary with the amount of cost overstatement*. The reason for this is quite simple: both p and s vary with the product $\delta C(Q)$, the same as the amount of tax evaded, $t\delta C(Q)$. Since $\partial EU/\partial\delta$ must be zero at the optimum, so is the component of $\partial EU/\partial Q$ derived from any function of $\delta C(Q)$. We are thus left with equating to zero the expected marginal utility of legitimate net profits.

Suppose, alternatively, that the monopolists decides on the overstated *amount*, G, where $0 \leq G \leq R(Q)-C(Q)$, rather than on the overstatement proportion. Its problem then becomes

$$EU = [1 - p(G)]U(\pi_1) + p(G)U(\pi_2) \tag{1'}$$

where $\pi_1 = (1-t)[R(Q) - C(Q)] + tG$ (2')

and $\pi_2 = \pi_1 - s(G)tG.$ (3')

The first-order conditions for an interior maximum change to

$$\frac{\partial EU}{\partial Q} = (R' - C')(1-t)[(1-p)U'(\pi_1) + pU'(\pi_2)] = 0 \tag{4'}$$

and

$$\frac{\partial EU}{\partial G} = (1-p)tU'(\pi_1) + p(1-s-s'G)tU'(\pi_2) - p'[U(\pi_1) - U(\pi_2)] = 0, \tag{5'}$$

implying that $R' = C'$, *irrespective of whether (5') holds*. Hence, the output decision is independent of tax evasion as well as of the profit tax rate even if evasion is non-optimal or if the monopolist best choice is a corner solution where he evades his entire profit taxes (i.e., if $\partial EU/\partial G < 0$ everywhere).

The separability of the firm's real activity level from its tax evasion behavior is not exclusive to profit taxes. Marrelli (1984) and Yaniv (1988) have shown that the firm's output and employment decisions are separable of sales tax evasion and withholding tax evasion, respectively. In a forthcoming note, Yaniv (1995) develops a general model of tax evasion applicable to any type of tax that might be evaded by the firm (profit, sales, payroll, withholding, etc.). The model shows, inter alia, that the firm's activity level is *always* separable from its evasion decision, irrespective of the type of tax evaded or of whether the firm is competitive or monopolistic. A basic feature of the model is that the probability of detection and the penalty rate are exogenously fixed. The present discussion suggests that these restrictions may not be necessary.

Indeed, viewing G as the statement deviation of the firm from the true value of *any* tax base, eq. (4') implies that the firm's real activity level, determining the tax base, is actually derived as if the firm were maximizing the expected utility of its legitimate net profits, ignoring its evasion decision and irrespective of its particular activity variable.

ENDNOTES

[1]Wang writes the first-order conditions as follows:

$$\frac{\partial EU}{\partial Q} = (1-p)[(R'- C')(1-t) + t\delta C']U'(\pi_1) - \delta p'C'U(\pi_1) +$$

$$p[(R'- C')(1-t) + (1-s-s'\delta C)t\delta C']U'(\pi_2) + \delta p'C'U(\pi_2) = 0$$

$$\frac{\partial EU}{\partial \delta} = (1-p)tCU'(\pi_1) - p'CU(\pi_1) +$$

$$p(1-s-s'\delta C)tCU'(\pi_2) + p'CU(\pi_2) = 0.$$

Rearranging terms, (4) and (5) will be obtained. However, assuming apparently that separability cannot be expected out of the above expressions, Wang retreats to examinimg the implications of profit tax evasion on monopolistic output, concluding that everything is possible.

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